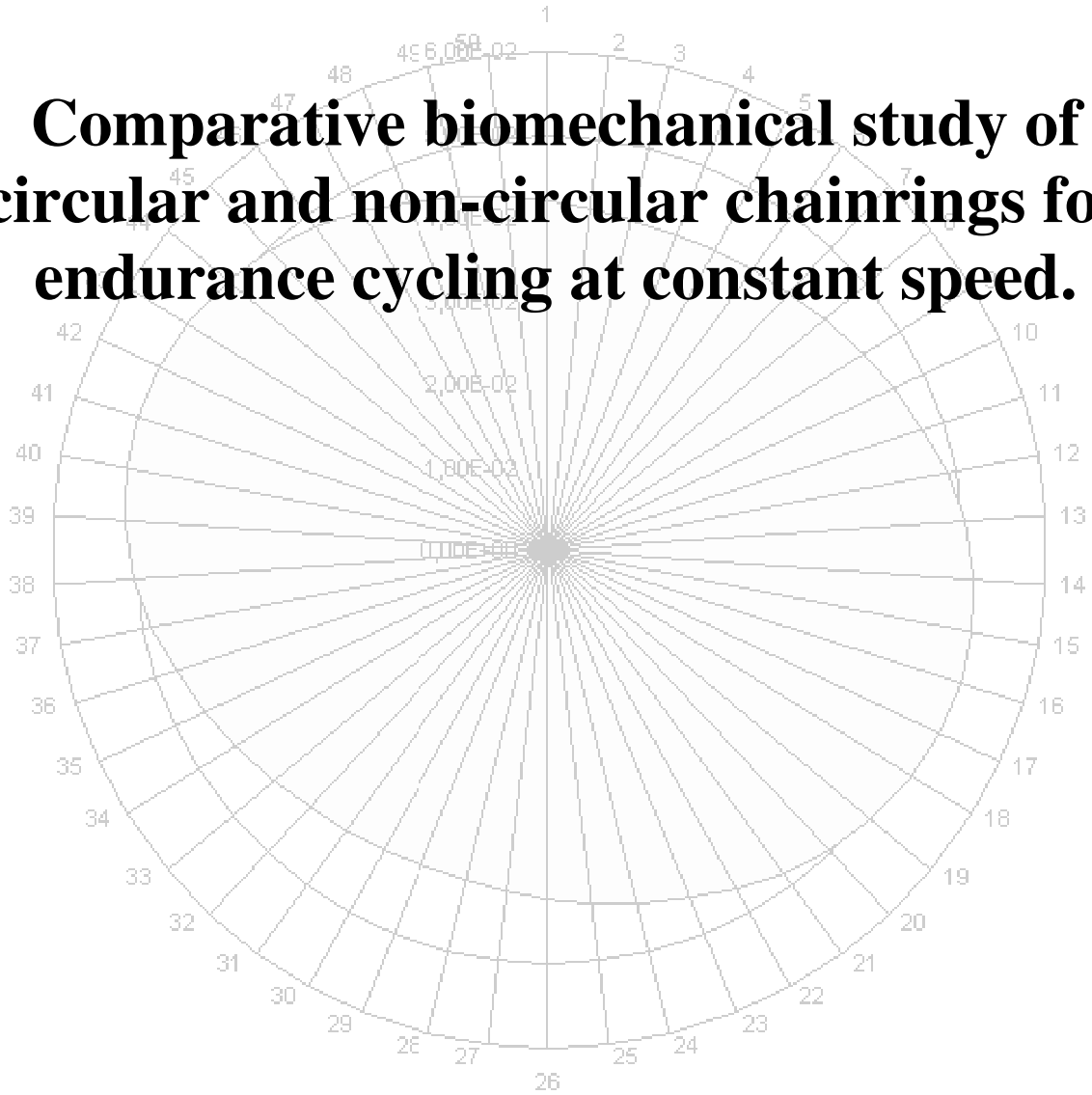


Comparative biomechanical study of circular and non-circular chainrings for endurance cycling at constant speed.



**L. Malfait, M.Mech.Eng.
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Release 2 , May 2010

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Abstract – Non-circular chainrings have been available in cycling since the 1890's. More recently, Shimano's Biopace disaster has spoiled the market for oval chainrings. The Harmonic (1994) was re-launched in 2004 under the brand name O.symmetric with some important successes in professional cycling. In 2005, the Q-Ring (Rotor) entered the cycling scene. However, non-circular chainwheels have not yet conquered the cycling world. There are many reasons for this: the conservative world of cycling, the suffocating market domination of an important manufacturer (and sponsor) of circular chainrings, the difficult bio-dynamics not understood by users and last but not least, it is not easy to measure and to prove the advantages of non-circular versus circular. Any reasonable non-circular chainwheel has about 50% chance of being better than the circular shape. The only question is: what is the optimum shape and how large can the difference be? The objective of this paper is to compare different chainring designs. Relying on a mathematical model a biomechanical comparison was made between circular and non-circular chainrings. The results of the study indicate clearly that (Criterion 1) for equal crank power for both circular and non-circular chainwheels, the peak joint power loads can be influenced favourably by using non-circular designs. For equal joint moments (Criterion 2) for both circular and non-circular designs, the model calculates differences in total crank power and differences in peak joint power loads. Results for both criteria are mostly concurrent. The analysis also indicates that shape as well as ovality, but also orientation of the crank relative to the chainring are important parameters for optimum design. It was found that some non-circular shapes are clearly better than other designs. The mathematical model can also be used as a tool for design optimization. Besides the commercial available non-circular chainrings, some 'academic' non-circular profiles were investigated.

Release 2 differs from the previous publication by the use of the MATLAB® software package for the mathematical model in stead of programs developed in Pascal. Conclusions in release 2 completely confirm the findings from the first release, although with more moderate crank power efficiency gains. In release 2, result tables are replaced by graphs.

1. Introduction

In cycling, the bicycle-rider system can be modelled as a planar five-bar linkage.

See figure 1: Five bar linkage model of the bicycle-rider system.

The links are: the thigh, the shank, the foot, the crank and the linkage crank axis-hip joint.

The five pivot points are: the crank axis, the pedal spindle, the ankle joint, the knee joint and the hip joint.

Two pivot points are considered as fixed: the crank axis and the hip joint.

See among others Redfield and Hull, Journal of Biodynamics, vol 19-1986a, pages 317-329.

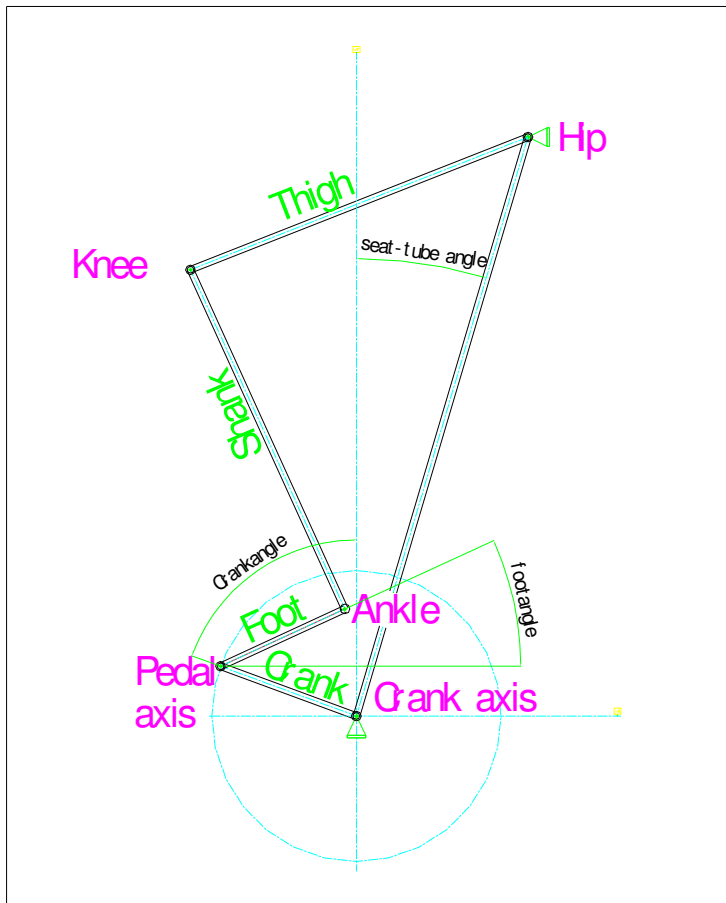


Figure 1: Five bar linkage model of the bicycle-rider system.

For a five-bar linkage, two kinematic variables are necessary to uniquely specify the linkage motion. Then the entire system is kinematically defined.

Usually the two kinematic variables are the crank angle and the pedal angle.

Hull et al experimentally measured the relationship between the crank angle and the angle of the pedal. They expressed the pedal angle (= angle of the foot) as a function of the crank angle.

A relatively accurate representation of this function is a sine function of the form

$$\text{Pedal angle } (\Delta) = A_1 + A_2 * \sin (\alpha + A_3)$$

where α is the crank angle

and A_1 , A_2 and A_3 are constants experimentally determined.

Using the work of Bolourchi and Hull, we valued

$$A_1 = 20.76^\circ$$

$$A_2 = 22.00^\circ$$

$$A_3 = 190.00^\circ$$

Additional information and input data are to be specified:

- Position of the hip axis versus the crank axis (defined by the seat height and the seat tube angle).
- Length of the bars: crank arm, foot, shank, thigh and the linkage crank axis-hip joint.
- Relative position of the centre of gravity of the foot, shank and thigh versus their pivot points. (*)
- Mass of foot, shank and thigh. (*)
- Moments of inertia of foot, shank and thigh. (*)
- Crank angular velocity.

(*) Values of the anthropometric parameters were estimated using the work of Dempster, Whitsett and Dapena.

This study assumes a cadence of 90 crank revolutions per minute. Hence cycle time is 0.667 sec.

This pedalling rate is generally accepted as being optimal (Hull et al) and preferred by trained endurance cyclists.

The research also assumes a constant speed of the bicycle, which means a constant chain linear velocity.

As a consequence a circular chainring has a constant angular velocity of the crank throughout one revolution.

Non-circular chainrings have variations in angular crank velocity during one crank cycle: this means, the crank angular velocity is a function of time.

The relation 'crank angular velocity as a function of time' for non-circular chainrings must be known and will be investigated later.

The assumption is made that the forces, developed in the muscles of the lower limbs, are directly related to the moments in the joints ('joint-torques'): ankle, knee and hip moments respectively.

Further in this paper, a method of calculation will be developed and presented which enables determination of the moments ('torques') and power in the joints as a function of the cycle-time.

2. Moments in the joints.

In order to develop a well-defined force on the pedal, the related muscles of the joints have to develop in each of the joints a well-defined joint moment (joint torque).

The force on the pedal (pedal force vector) varies in magnitude and in orientation as a function of the crank angle.

By means of a pedal dynamometer, the normal and tangential components of the pedal force were measured and registered as a function of time. These measurements were executed at constant (steady state) crank cadence (90 rpm) and at constant power level (200 W), see figure 2.

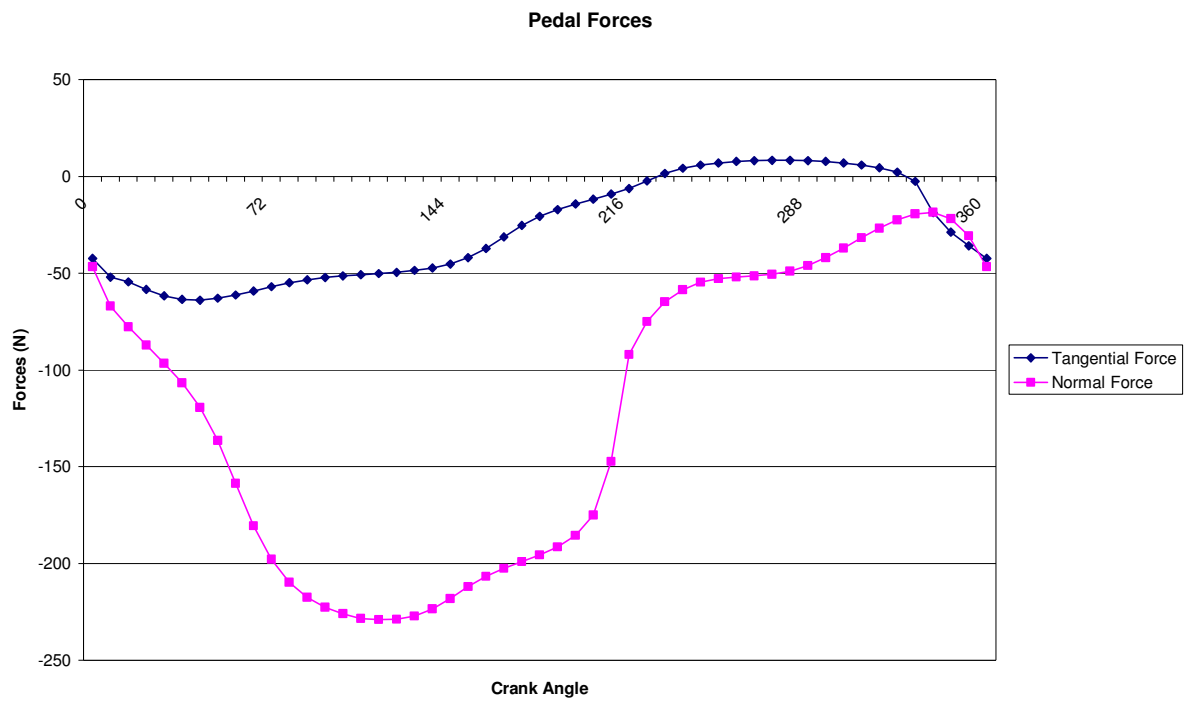


Figure 2: Measured tangential and normal pedal forces (Hull et al)

Given the measured normal and tangential pedal forces and taking into account the known relationship of the pedal angle as a function of the crank angle, the normal and tangential crank force as a function of the crank angle can be calculated.

The tangential crank force delivers the crank moment and consequently the crank power.

See figure 3: Relationship between tangential and normal pedal forces and crank forces.

Using vector decomposition techniques the force vector on the pedal can be decomposed into horizontal (X) and vertical (Y) direction respectively:

F_x : denotes the horizontal pedal force component

F_y : denotes the vertical pedal force component

Both pedal force components, together with the dynamic forces and the moments of the limbs are used to calculate the joint moments.

See figure 4: Pedal force vector decomposition: tangential and normal, F_x , F_y .

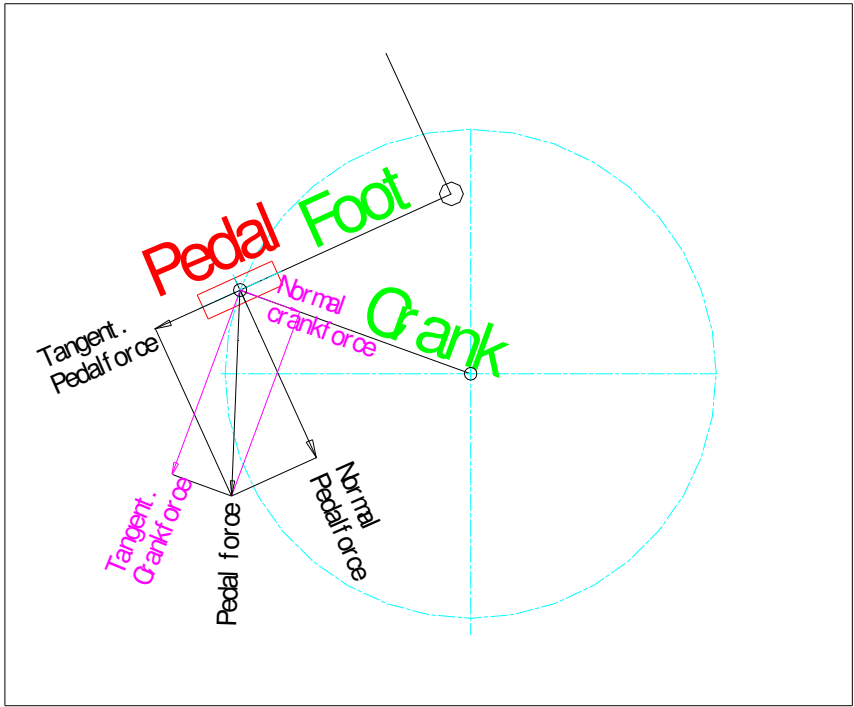


Figure 3: Relationship between tangential and normal pedal forces
Relationship between tangential and normal crank forces

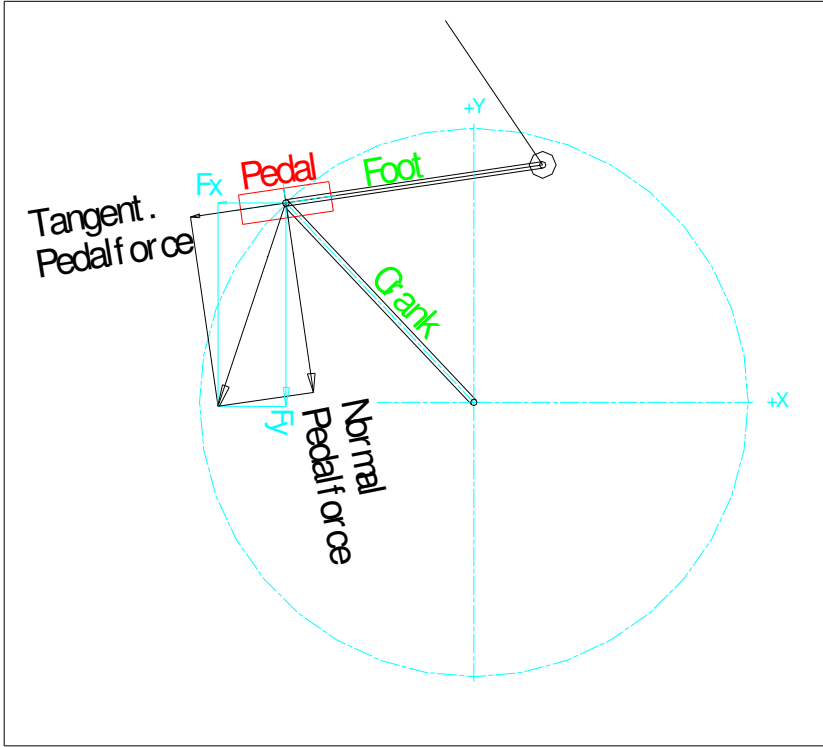


Figure 4: Pedal force vector decomposition: tangential and normal, F_x , F_y .

By means of inverse dynamics the joint forces and the joint moments were calculated, ref. figure 5.

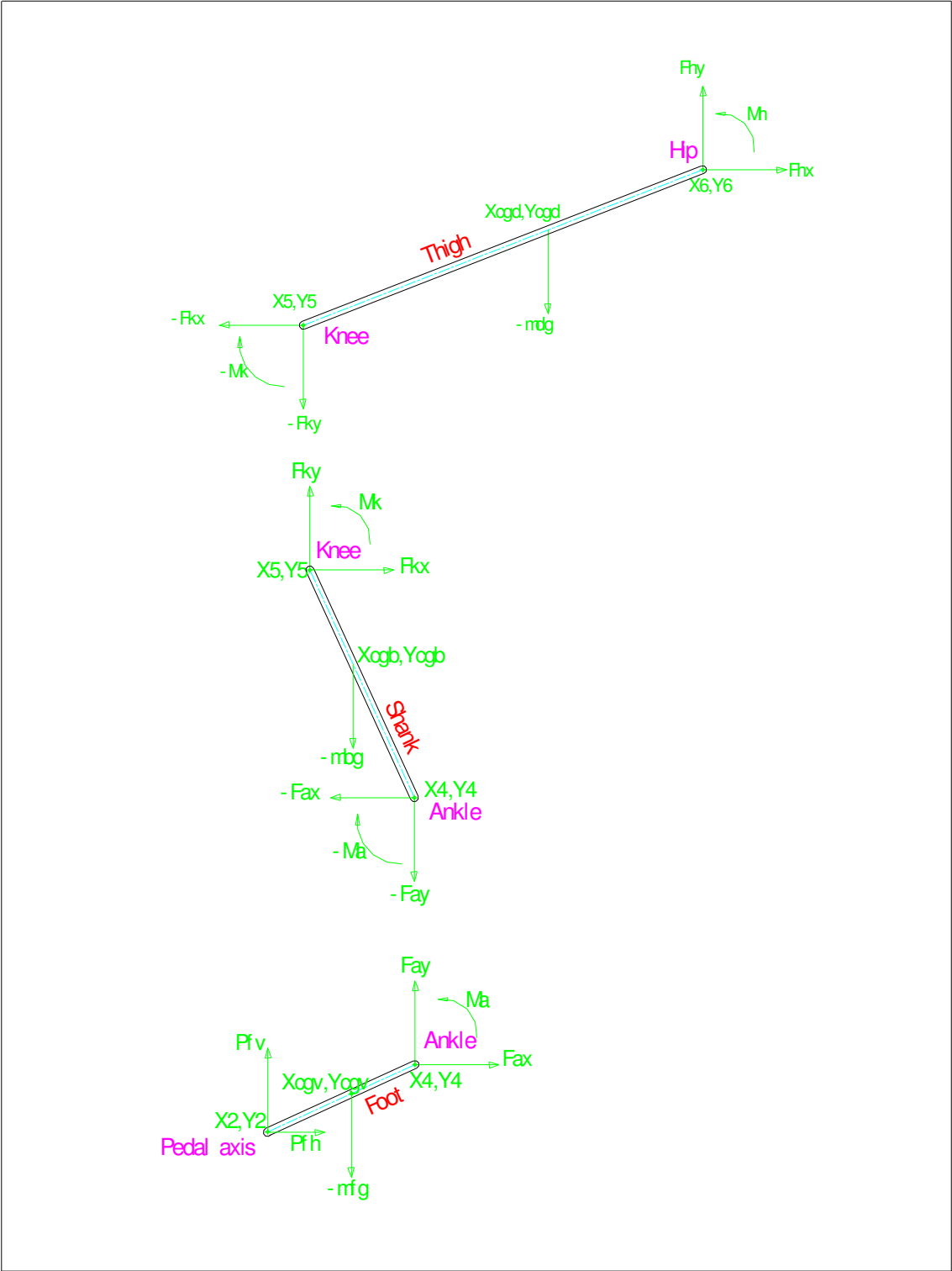


Figure 5: Free body diagrams of each link: balances of forces and moments.

$$P_{fv} = -F_y$$

$$P_{fh} = -F_x$$

The position data of each of the joints were calculated as a function of time. Given the calculated position data of each of the limbs (links):

by taking the first derivative

- the linear velocity of the centre of gravity in X and Y
 - the angular velocity of the limbs
 - the angular velocity of the joints
- were calculated.

by taking the second derivative

- the linear accelerations in X and Y
 - the angular accelerations
- were calculated.

From the free-body diagram (see figure 5) the reaction forces in the joints and the joint moments can be calculated for the ankle, the knee and finally for the hip.

For the foot with ankle joint:

$$F_{ax} = m_f * aX_{cgf} - Pf_h$$

$$F_{ay} = m_f * aY_{cgf} - Pf_v + m_f * g$$

$$M_a = I_f * a_{footangle} - Pf_v * (X_4 - X_2) + Pf_h * (Y_4 - Y_2) - F_{ay} * (X_4 - X_{cgf}) + F_{ax} * (Y_4 - Y_{cgf})$$

Nomenclature:

M_a = ankle moment.

I_f = moment of inertia of the foot about the centre of gravity.

$a_{footangle}$ = angular acceleration of the foot.

Pf_v = reaction force at the pedal, vertical component.

Pf_h = reaction force at the pedal, horizontal component.

X_2, Y_2 = coordinates of pedal spindle.

X_4, Y_4 = coordinates of ankle axis.

X_{cgf}, Y_{cgf} = coordinates of the centre of gravity of the foot.

F_{ax} = force at the ankle joint, X component.

F_{ay} = force at the ankle joint, Y component.

m_f = mass of the foot.

aX_{cgf} = linear acceleration of the centre of gravity of the foot, X component.

aY_{cgf} = linear acceleration of the centre of gravity of the foot, Y component.

g = acceleration of gravity (9.81 m/s²)

Similar equations are defined for the shank with the knee joint and for the thigh with the hip joint.

The total joint moments given in the equations above are determined by the static forces and dynamic forces and the respective moments.

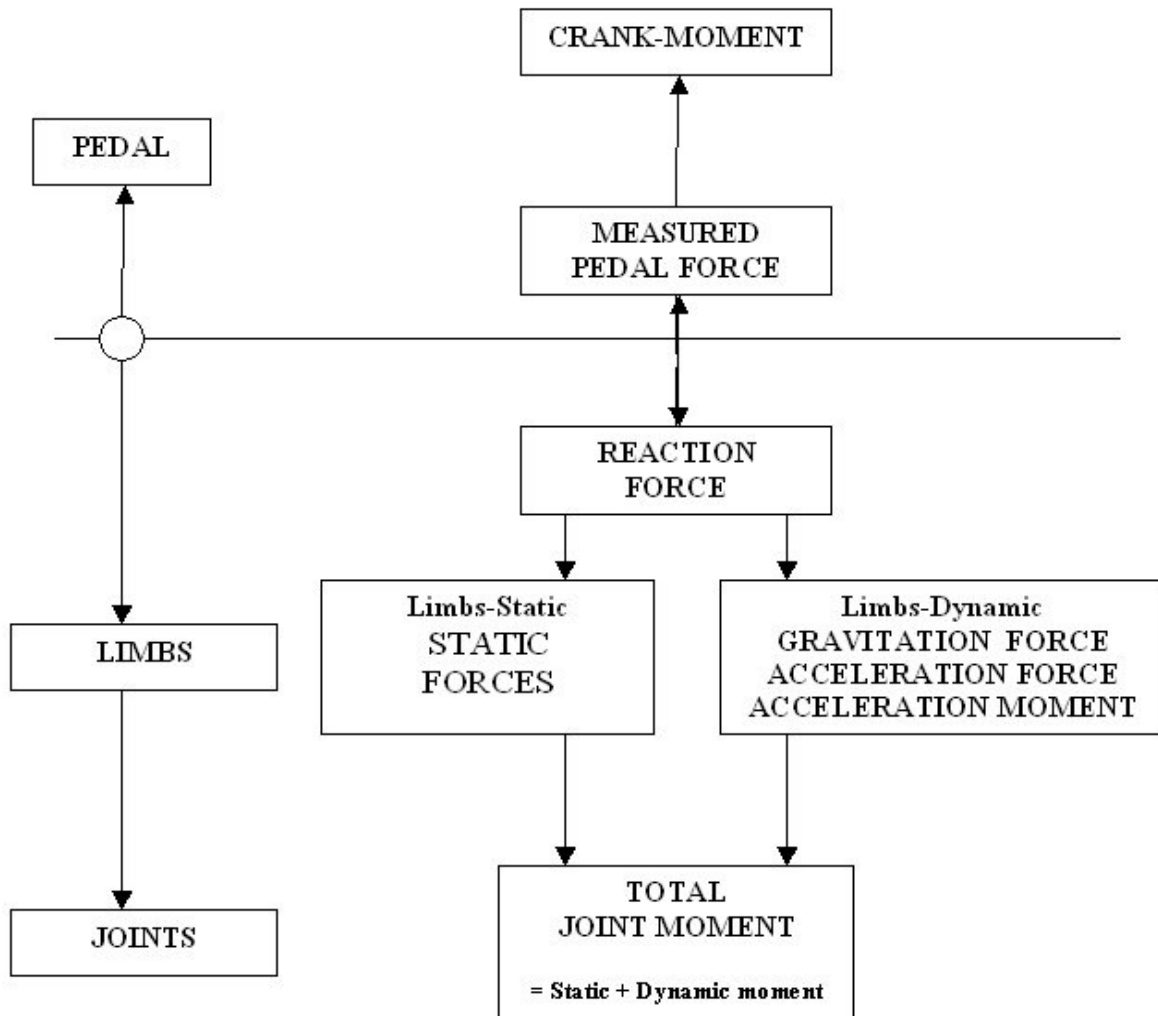


Figure 6 shows the schematic of this partitioning providing a valuable insight into the dynamics of the pedalling process.

$$\text{Joint power} = \text{Joint moment} * \text{Joint angular velocity}$$

Joint angular velocity is a function of crank angular velocity.

As a consequence, for a given bar geometry, given anthropometric data and a given pedalling rate, the dynamic forces (-moments) can be influenced by varying the crank angular velocity.

The variation of the crank angular velocity changes the dynamic joint moments, via the dynamic forces and –moments of the leg segments (limbs). These

changes affects the instantaneous total joint moment and the instantaneous total joint power.

This insight leads to the conclusion that changing the crank angular velocity gives the opportunity to introduce possible improvements to the drive mechanism of the bicycle.

Taking a constant pedalling rate for both means that:

- the circular chainring has a constant crank angular velocity
- the non-circular chainring has a varying crank angular velocity, which is defined by the geometry of the chainring.

3. Method to calculate the crank angular velocity and crank angular acceleration for any chainring geometry.

The chainring geometry has to meet the following conditions:

1. The contour of the pitch-polygon (pitch-curve) must be equal to n times the pitch of the chain, whereof n equals the number of chainring teeth.
2. Each side of the pitch-polygon must be exactly equal to the pitch of the chain.
3. The chainring geometry must be convex; this means no concave sections are allowed.
4. Point-symmetry for the pitch-curve is a minimum condition.

Constructive limitations may also arise because of the front derailleur: the ratio major axis versus minor axis (ovality) of a non-circular chainring has to be kept within certain limits.

In order to define the crank angular velocity as a function of **time** for a non-circular chainring, the combination chainring-chain must be considered. This remains also the case even when the pitch-curve is mathematically defined e.g. for an ellipse.

The procedure to follow is visualised in figure 7.

Keeping the ‘working chain length’ constant, the successive positions of the chainring were drawn (using AutoCAD software), each time corresponding with one tooth rotation of the sprocket. One tooth rotation also equals one time unit (constant speed of the bicycle is assumed).

In case the curving of the chainring is not constant (non-circular), a deviation versus the theoretical angle of rotation was measured (a kind of ‘interference’). Applying this method for each chainring tooth, a matrix with crank angle positions and corresponding time can be created.

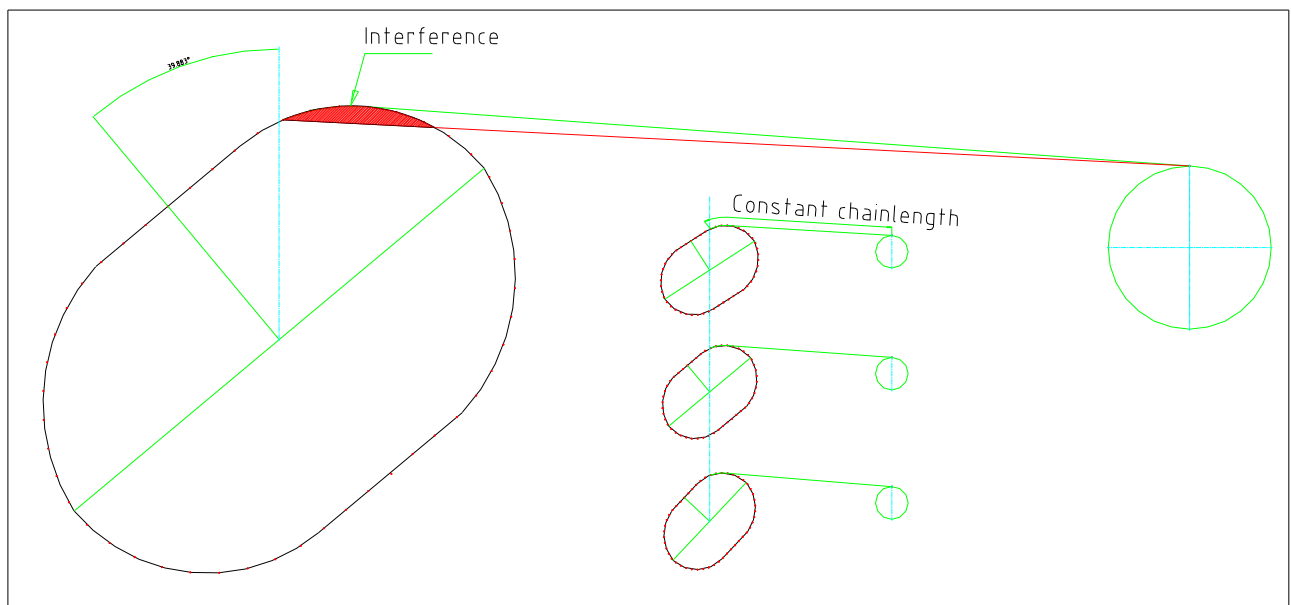


Figure 7: How to measure the crank angle per unit of time.

By means of curve-fitting techniques (polynomial regression) the most optimal mathematical expression of the crank angle as a function of time is calculated.

In general, a nine degree polynomial of the form

$Y = A_0 + A_1 * X + A_2 * X^2 + A_3 * X^3 + \dots + A_N * X^N$ with $N = 9$ fits closely to the data (correlation > 0.9999).

By taking the first derivative of the equation, with respect to time, the relationship “crank angular velocity as a function of time” is determined.

By taking the second derivative with respect to time, the relationship ‘crank angular acceleration as a function of time’ is determined.

4. Criteria of bio-mechanical comparison circular with non-circular chainrings.

The mathematical model was programmed using MATLAB® software. The MATLAB® symbolic math toolbox generates and calculates all the necessary first and second derivatives.

The ten coefficients of the polynomial equation of the crank angle as a function of time are used as input data.

Antropometric, geometric, and other data are constants stored in the MATLAB® files and are adaptable if needed.

The circular chainring is considered as being the reference.

Data of standard pedal forces as a function of time are applied in case of the circular chainring.

The pedal force profile measured by Hull et al is used (see figure 2). This approach is acceptable for a comparative study.

All chainrings considered in the study are “normalised” in AutoCAD to 50 teeth.

Given the above-mentioned data, by means of MATLAB®, the mathematical model calculates the moments and the power as well as further outputs.

All important output data are represented in graphs, using MATLAB® graphic tools.

Criterion 1:

Given the same instantaneous crank-power development throughout the full crank cycle for both circular and non-circular chainrings, the development of the joint-power was calculated for both circular and non-circular designs. Calculations were executed for the knee and the hip joint.

During the first part of the cycle, where the joint angular velocity is positive, the extensor muscles of the joints are the main drivers. For the hip this is mainly the Gluteus Maximus and for the knee primarily the Rectus Femoris and the Vastii.

During the second part of the cycle, where the joint angular velocity is negative, the flexor muscles of the joints are the main drivers. For the hip this is mainly the Rectus Femoris and for the knee primarily the Gastrocnemius, the Biceps Femoris and the Hamstrings.

Approach:

In a first run, the MATLAB® program calculates the positions, the velocities, the accelerations, the total joint moments, the total joint power and the crank power of the circular chainring.

The pedal reaction forces in the X and Y directions were calculated using the normal and tangential pedal forces measured by Hull et al (see figure 2).

Time was declared as a symbolic variable, so all subsequent equations were evaluated as a function of time.

A graph of the knee power and the hip power development as a function of time was prepared.

In a second run, MATLAB® calculates the crank angular velocity of the non-circular chainring being the first derivate of the crank angle versus time. The crank power as a function of time, taken over from the first run (circular chainring), was now used as input. From this, the pedal reaction forces in the X and Y directions were deduced, assuming that these reaction forces relate to each other in the same way as the X and Y components of the circular chainring do. Knee power and hip power versus time were now recalculated. Plots of knee power and hip power versus time for both the circular and non circular chainring, are presented.

Criterion 2:

Comparison of the total crank power over the full crank cycle taking into account identical development of the instantaneous joint-moments for both, circular and non-circular chainring..

Approach:

In a first run, the MATLAB® program calculates the positions, velocities, accelerations, total joint-moments, total joint power and crank power of the circular chainring. Time is declared as a symbolic variable, so all subsequent equations are evaluated as a function of time. A graph of the crank power as function of time is prepared.

In a second run, MATLAB® calculates the crank angular velocity of the non-circular chainring, being the first derivate of the crank angle versus time. The joint-moments as function of time, taken over from the first run are now used to recalculate the X and Y components of the crank force. Here from, crank power versus time is calculated and a plot of crank power versus time is presented. The graphs are clearly showing the differences in crank power development between the circular and the compared non-circular chainring. A MATLAB® tool allows to calculate and to display the mean value of the crank power over one full cycle for one pedal, for both the circular and the non circular chainring.

5. Non-circular chainring types

Convention: 1. Crank angle

* crank arm vertical equals 0° ,

arbitrary defined as being “Top-Dead-Centre” (T.D.C.)

*rotation: counter clockwise

***crank angle** is being measured from T.D.C

(= crank arm vertical), counter clockwise, to major axis.

2. Ovality (‘e’ in the figures): ratio of major axis to minor axis

- O.symmetric-Harmonic
 - designed: 1993
 - inventors: J.L. Talo & M. Sassi, France
 - ovality: 1.215
 - geometry: see figure 8
 - symmetry: point symmetric (bi-radial)
 - chainring radius proportional with variation of crank torque
 - angle major axis versus crank arm: 78° (major axis assumed to be the middle of the circle segment of the oval);
 - radial oriented chainring teeth
 - commercialised

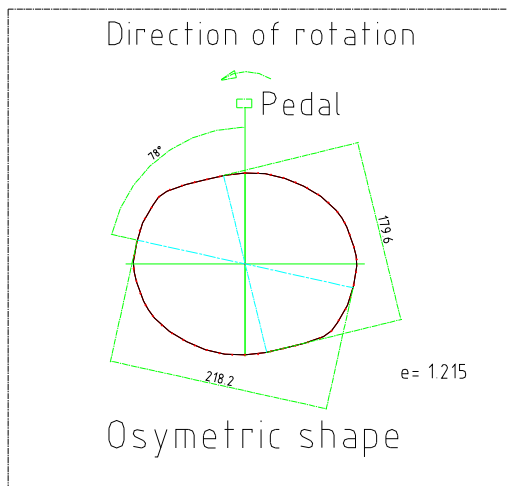


Figure 8: O.symmetric-Harmonic

- Hull oval
 - designed: 1991
 - inventor: prof M.L. Hull, Univ California, Davis, USA
 - ovality: 1.55
 - geometry: see figure 9
 - symmetry: point symmetric (bi-radial)

- theoretical shape to eliminate “internal work”
- angle major axis vs crank arm: 90°
- not commercialised

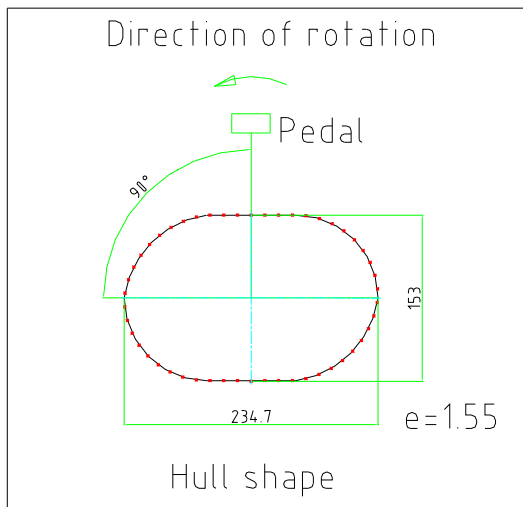


Figure 9: Hull oval

- Rasmussen oval
 - designed: 2006
 - inventor: prof John Rasmussen, Univ of Aalborg, Denmark
 - ovality: 1.30
 - geometry: ellipse-like, see figure 10
 - symmetry: bi-axis symmetric
 - designed to minimize maximum muscle activation
 - angle major axis vs crank arm: 72°
 - not commercialised

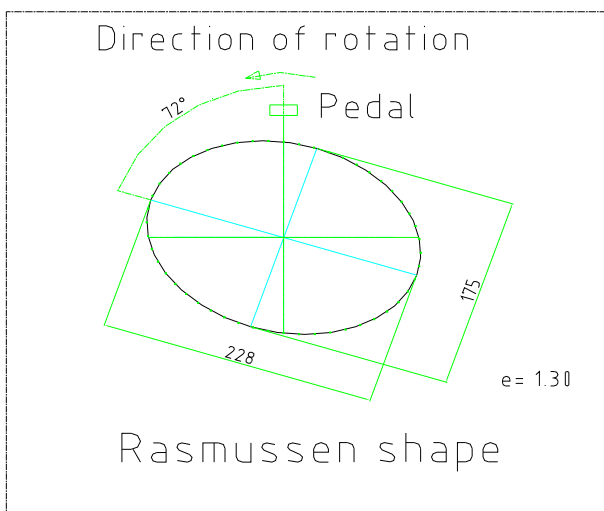


Figure 10: Rasmussen oval

- Q-Ring (Rotor)
 - designed: 2005
 - inventor: Pablo Carrasco, Rotorbike, Spain
 - ovality: 1.10
 - geometry: modified ellipse (circle arcs at extremities of major axis), see figure 11
 - symmetry: bi-axis symmetric
 - designed to minimize time spent in the dead spots and to maximize the benefit of the power stroke
 - angle major axis vs crank arm: adjustable, advised 70° - 75°
 - commercialised

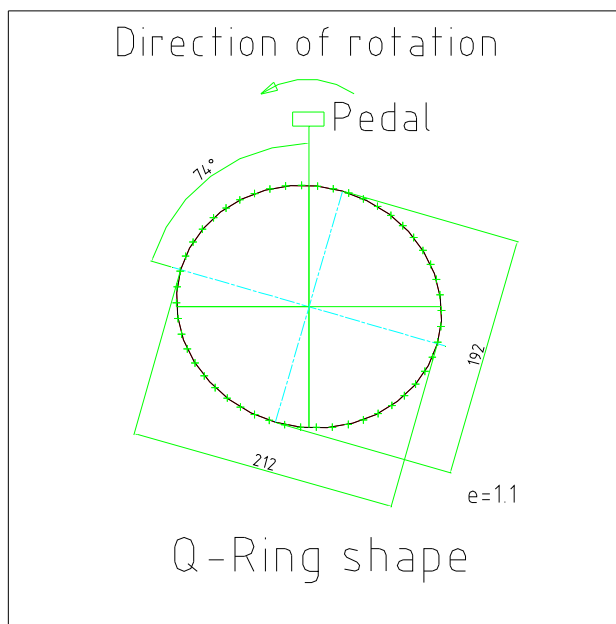


Figure 11: Q-Ring

- Biopace oval
 - designed: 1983
 - inventor: Shimano, Japan (Prof. Okajima)
 - ovality: 1.04 (earlier makes 1.09, 1.17...)
 - geometry: skewed ellipse with major and minor axes not perpendicular, see figure 12
 - symmetry: point symmetric
 - designed to take advantage of leg inertia
 - angle major axis vs crank arm: -8° (crank arm approximately parallel to major axis)
 - commercialised

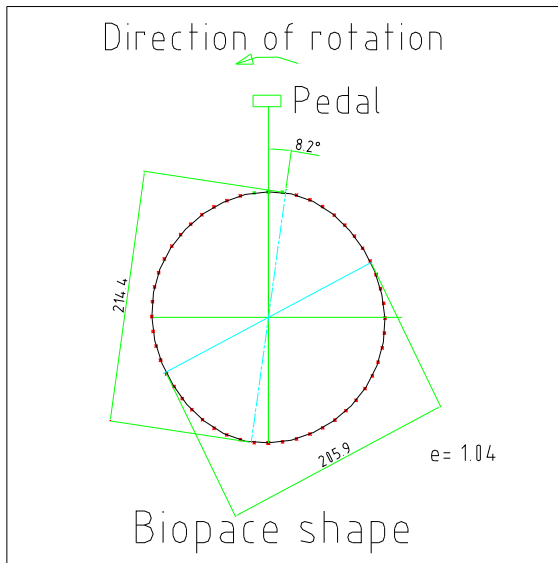


Figure 12: Biopace

- **OVUM ellipse**
 - designed: before 1980 (?)
 - inventor: ?
 - ovality: different types, 1.18 and 1.235
 - geometry: ellipse, see figure 13
 - symmetry: bi-axis symmetric
 - designed to reduce negative effects of dead spots
 - angle major axis vs crank arm: 90° (also types with adjustable crank orientation)
 - commercialised

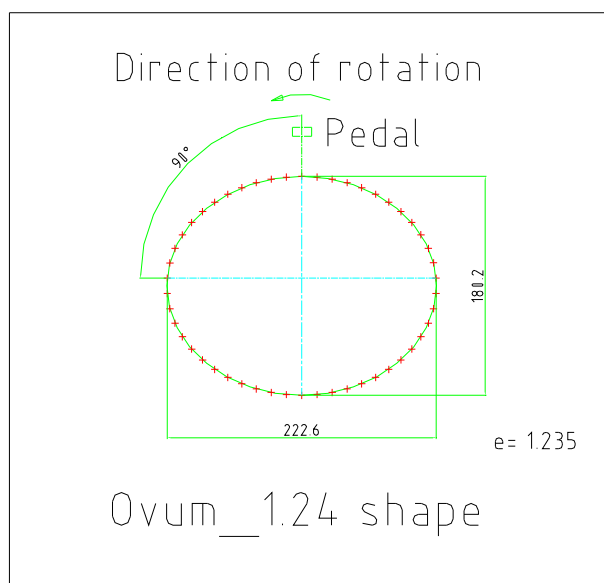


Figure 13: OVUM ellipse

- Ogival oval
 - designed: 1993
 - inventor: Bernard Rosset, France
 - ovality: 1.235
 - geometry: intersection of 2 circle arcs with circle centres on minor axis, see figure 14
 - symmetry: bi-axis symmetric
 - designed to reduce negative effects of dead spots and facilitate climbing
 - angle major axis vs crank arm: 54°
 - commercialised

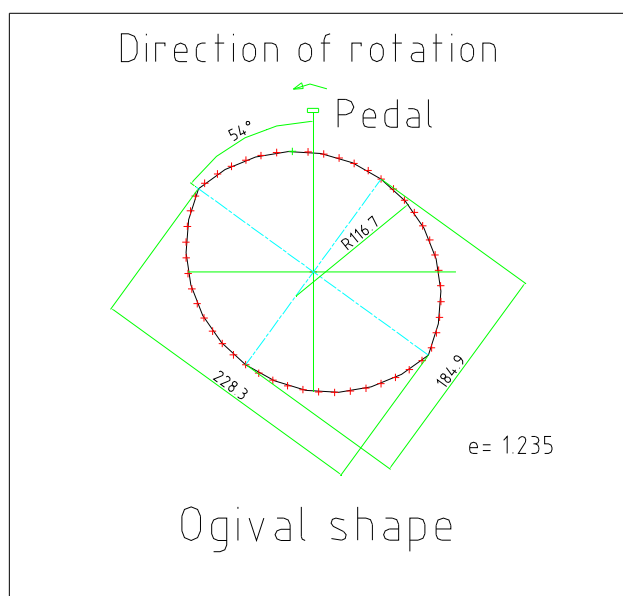


Figure 14: Ogival

- Polchlopek oval
 - designed: 1970 (?)
 - inventor: Edmond Polchlopek, France
 - ovality: 1.214
 - geometry: 2 semicircles joined by 2 bridges of 3 'flat' teeth, see figure 15
 - symmetry: bi-axis symmetric
 - designed to reduce negative effects of dead spots
 - angle major axis vs crank arm: 102°
 - commercialised

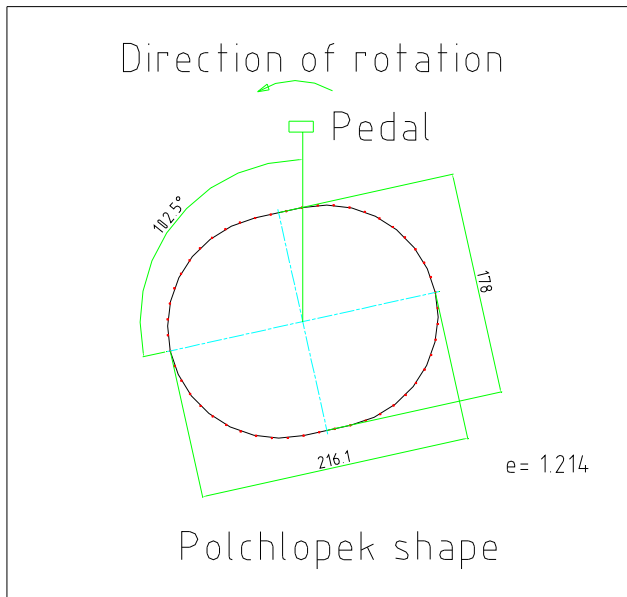


Figure 15: Polchlopek oval

- LM-Super oval
 - designed: 2009
 - inventor: Lievin Malfait
 - ovality: 1.31
 - geometry: see figure 16
 - symmetry: point symmetric (bi-radial)
 - chaining reflects polar plot of crank torque throughout one crank cycle
 - angle major axis versus crank arm: adjustable from 78° to 118° in 5 positions (major axis assumed to be the middle of the circle segment of the oval);
 - the 'flat teeth segment' and the "circle segment" are bridged by an Archimedean spiral segment.
 - chaining teeth perpendicular on the pitch-curve.
 - not commercialised

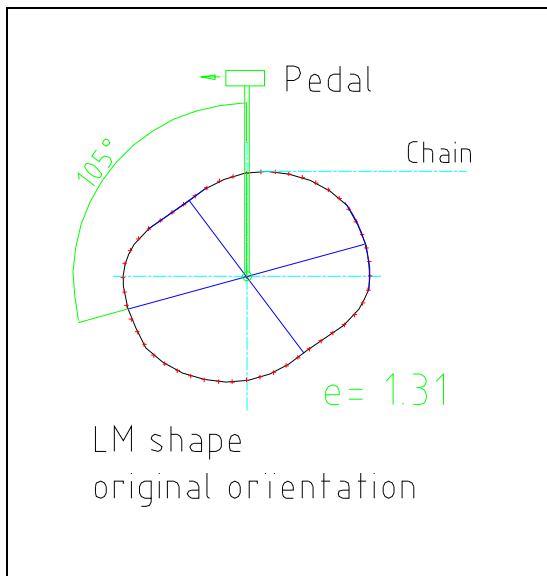


Figure 16: LM-Super oval

6. Biomechanical results

The properties and performances of the different non-circular chainrings examined in this paper, are displayed in the pictures and graphs below.

At the top of each page the pictures show the shape, the ovality and two different crank orientations of the chainring involved: on the left, the crank positioning proposed by the inventor or designer and, on the right, the optimal orientation calculated by this study.

In the middle of the page, the performances with respect to **criterion 2** are plotted (crank power development at equal joint moments, circular and non-circular).

The mean crank power of the non-circular chainring, calculated by MATLAB®, is plotted as a dash-dotted red line. A data tip indicates the value of the mean crank power of the non-circular.

The mean crank power of the circular chainring, also calculated by MATLAB®, is 104 W in all cases and is mentioned on the graphs.

The ratio between the mean non-circular crank power and the mean circular crank power is a measure for the efficiency gain of the non-circular chainring, compared to a circular one.

As an example: a ratio of 1.025 means that, at equal joint moments, the mean crank power of the non-circular chainring is 2.5 % superior versus the mean crank power of the circular, which is favourable.

At the bottom of the page, the graphs show the performances with respect to **criterion 1** (development of knee and hip power at equal crank power, circular and non-circular).

Data tips indicate the knee peak power (extensor muscles) for both the circular and non-circular chainring.

The ratio between the knee peak power (extensor muscles) of the non-circular chainring versus circular is a measure of the efficiency with respect to the knee joint peak load (extensor muscles).

As an example: a ratio of 0.94 (or 94%) means that, at equal crank power, the peak knee power (extensor muscles) is 6% inferior with a non-circular chainring compared to a circular one, which is favourable.

In cycling, the knee extensor muscles are assumed to be of major importance. Muscular fatigue and (knee) injuries primarily are caused by peak joint loads. Hence, comparing the knee peak power generated by the knee extensor muscles is useful and is a well-founded basis to compare and to judge the performances of non-circular chainrings.

Information about the development of the peak power loads in the knee flexor muscles, in the hip extensor muscles and in the hip flexor muscles, for both circular and non-circular chainring, can be found by reading out the different graphs (values are not mentioned in the overview tables).

6.1. Optimal crank orientation

By changing step-by-step the crank angle versus the major axis of the non-circular chainring we can search for an optimal crank orientation.

An optimal crank position would mean:

- the lowest peak power load in the joints, given the same crank power development (criterion 1)
 - the highest crank power efficiency, combined with the lowest peak power load in the extensor joint muscles of knee and hip, given the same joint moments (criterion 2)
- for both, circular and non-circular.

As an example we study the O.symmetric case.

versus circular chainring

| Angle Major Axis versus Crank | Criterion 1 | Criterion 2 | | |
|-------------------------------|--------------------------|----------------|--------------------------|-------------------------|
| | Peak Power Knee Extensor | Crank Power | Peak Power Knee Extensor | Peak Power Hip Extensor |
| 78.0° | -1.5% vs circ. | -0.7% vs circ. | -1.8% vs circ. | +4.6% vs circ. |
| 109.6° | -7.9% | +2.4% | -10.0% | +15.1% |
| 117.0° | -7.5% | +2.5% | -10.0% | +16.2% |
| 124.3° | -6.5% | +2.9% | -9.9% | +15.4% |
| 128.0° | -4.9% | +2.9% | -7.6% | +14.0% |
| 132.0° | -3.2% | +2.7% | -6.0% | +18.0% |

We notice the same trend for all the investigated chainring designs.

As a general rule we may conclude that the optimal crank orientation is located in the zone from 110° up to 120°, angle between major axis of the non-circular chainring and the crank, measured counterclockwise.

In case the crank is optimal oriented and the major axis of the non-circular chainring is vertical then we see the crank arm roughly perpendicular on the seat tube direction (“rule of thumb”).

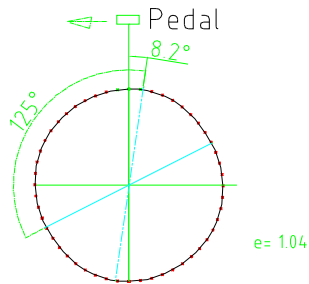
Above mentioned figures are applicable for a seat tube angle of about 73°.

The optimal crank orientation is a function of the bicycle geometry and the anthropometric parameters of the rider.

Simulations with a seat tube angle of e.g. 78° (time trial bike) learn that the optimal zone is located in the range between 105° and 115°.

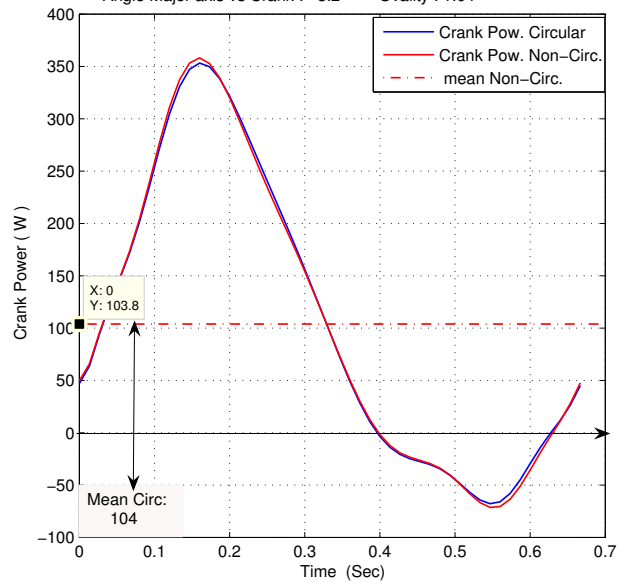
6.2. Graphs

Biopace Original -8.2°

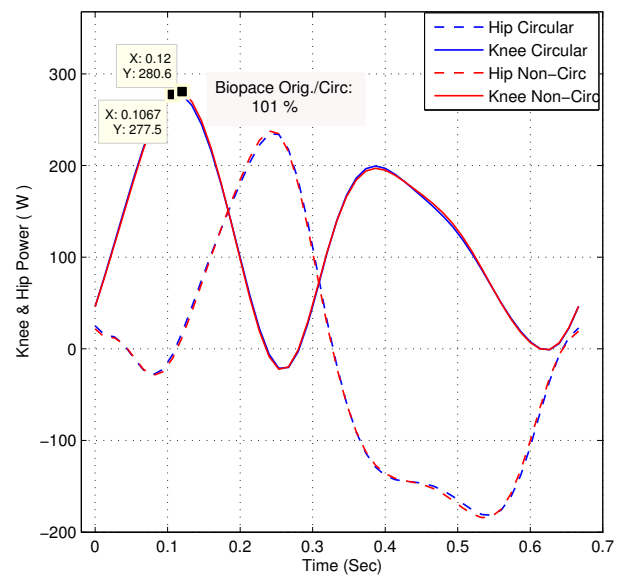


Biopace shape
Original orientation

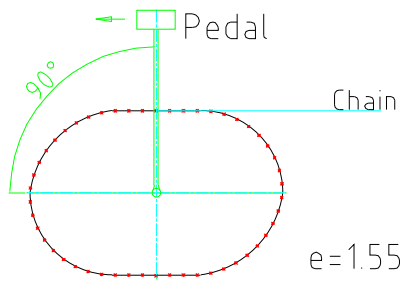
Crank Power – Circular vs biopace Orig., at equal Joint Moments --- 90 Rpm
Angle Major axis vs Crank : -8.2 ° -- Ovality : 1.04



Knee & Hip power Circular vs biopace Orig., at equal Crank Power --- 90 Rpm
Angle Major axis vs Crank : -8.2 ° -- Ovality 1.1

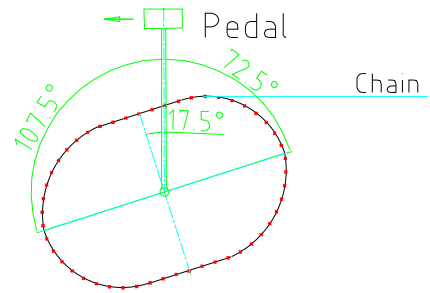


Hull Original 90°

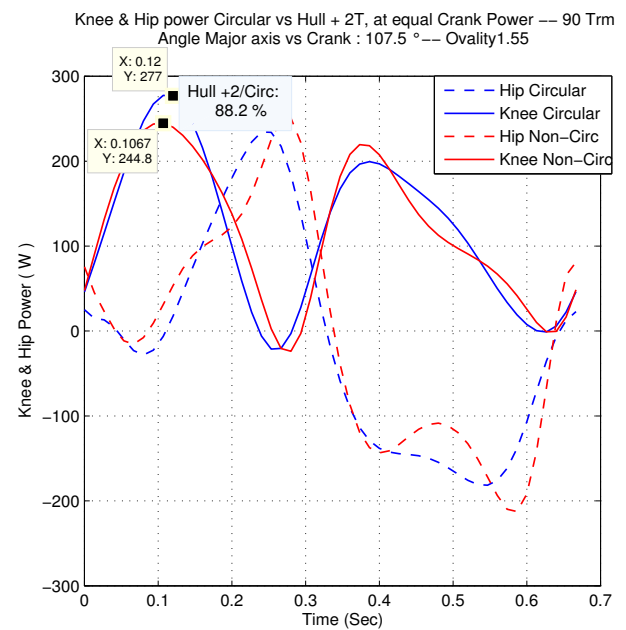
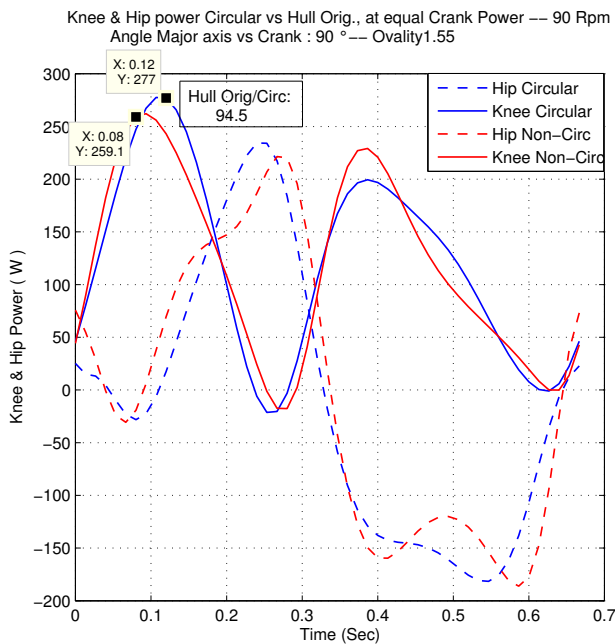
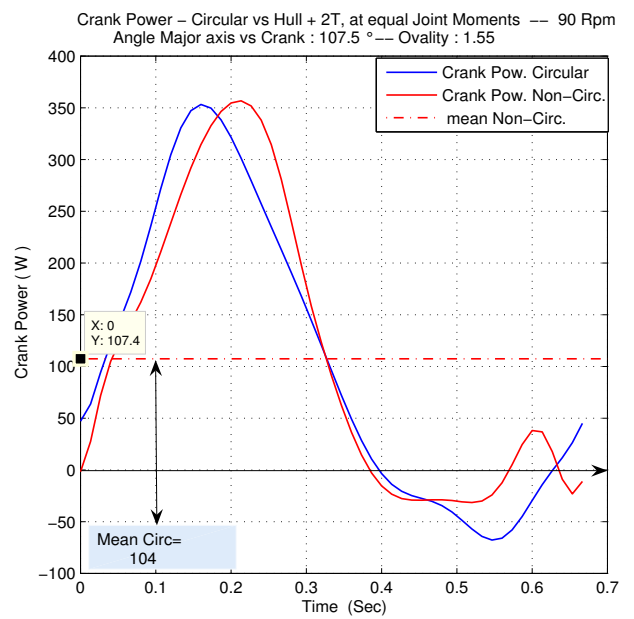
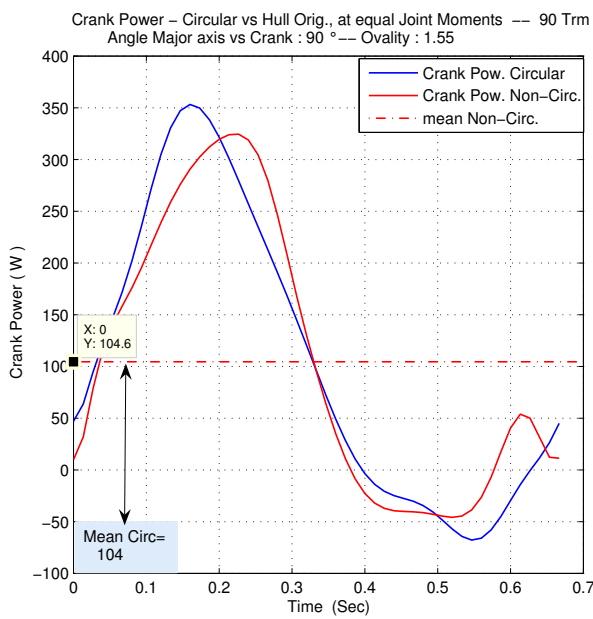


Hull shape original Orientation

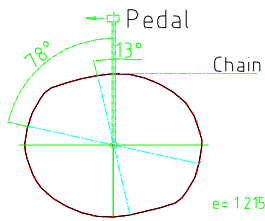
Hull Optimal 107°



Hull shape optimal Orientation

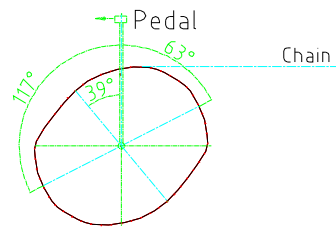


Osymmetric Original 78°



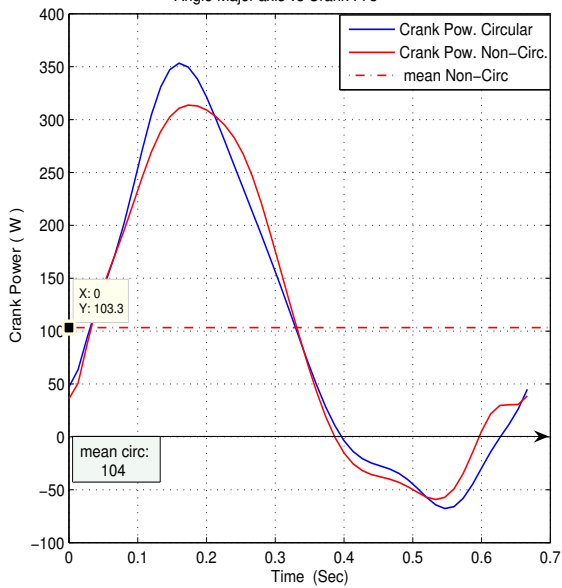
Osymmetric shape
Original orientation
according to patent

Osymmetric 117°

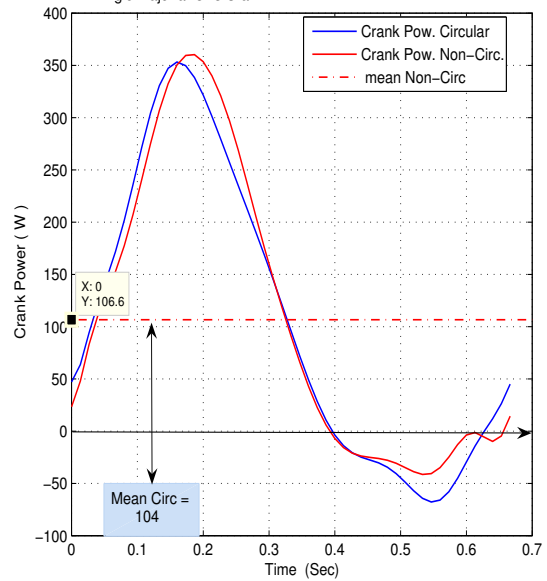


Osymmetric shape
Optimal orientation

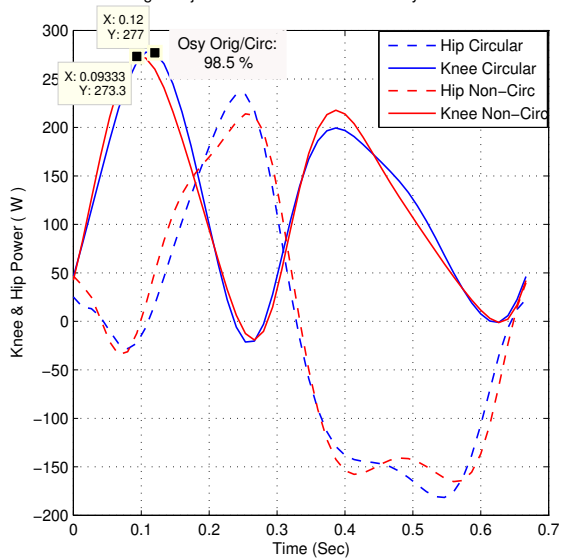
Crank Power – Circular vs Osymmetric original, at equal Joint Power --- 90 Rpm
Angle Major axis vs Crank : 78°



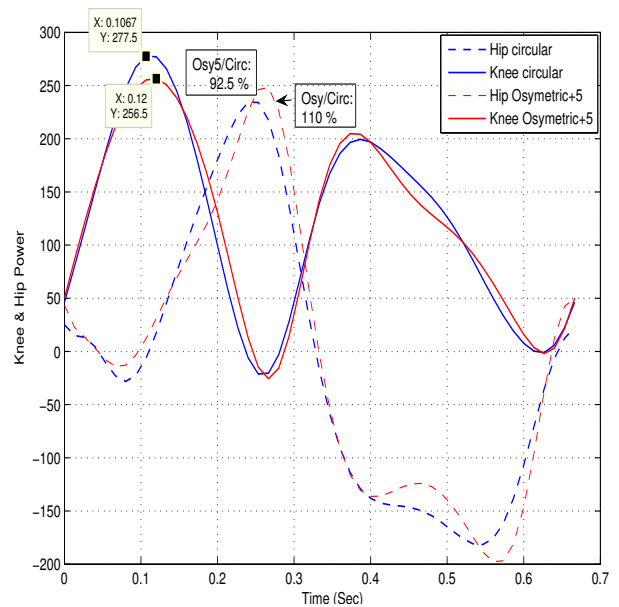
Crank Power – Circular vs Osymmetric +5T rotated, at equal Joint Moments --- 90 Rpm
Angle Major axis vs Crank : 117°



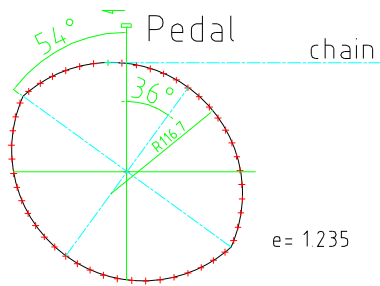
Knee & Hip power Circular vs Osymmetric original, at equal Crank Power --- 90 Rpm
Angle Major axis vs Crank : 78° --- Ovality 1.215



Knee & Hip power Circular vs Osymmetric+5t at equal Crankpower 90 Rpm
Angle Major axis vs Crank : 117°

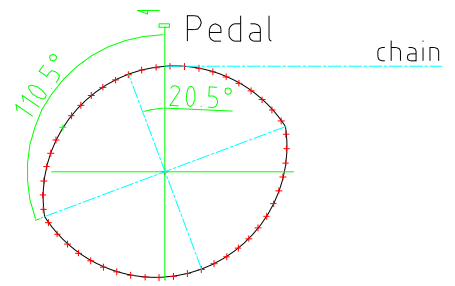


Ogival Original 54°

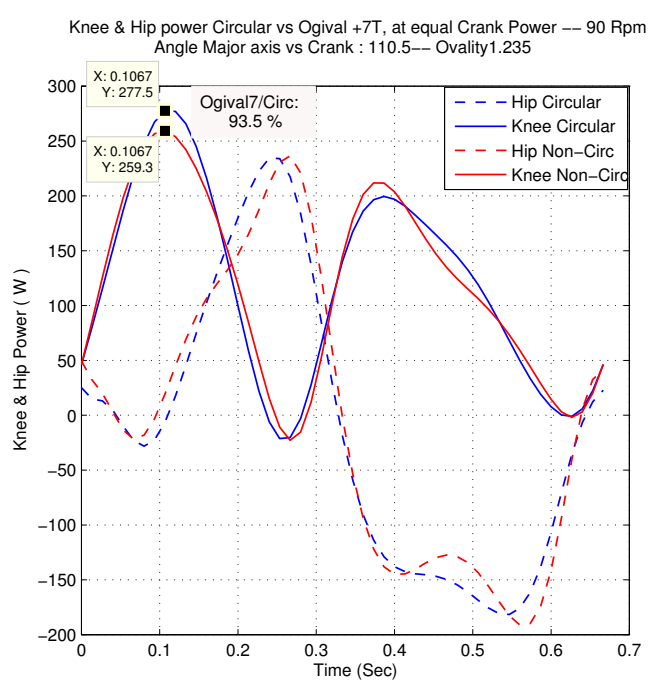
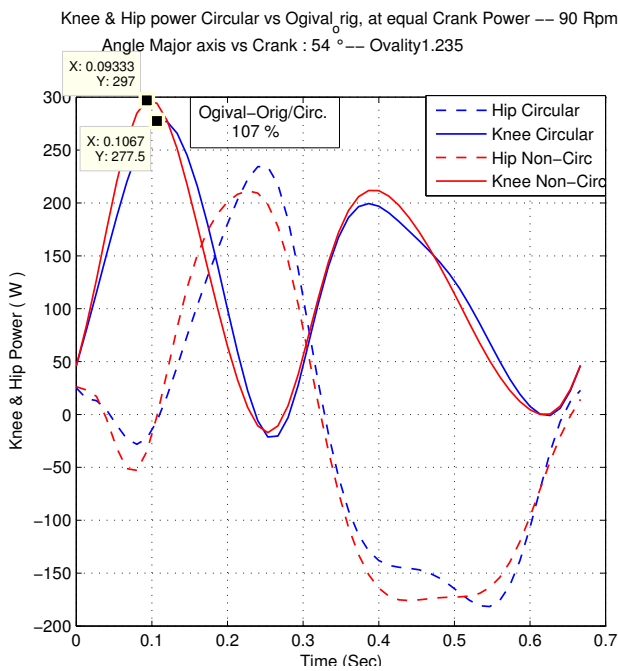
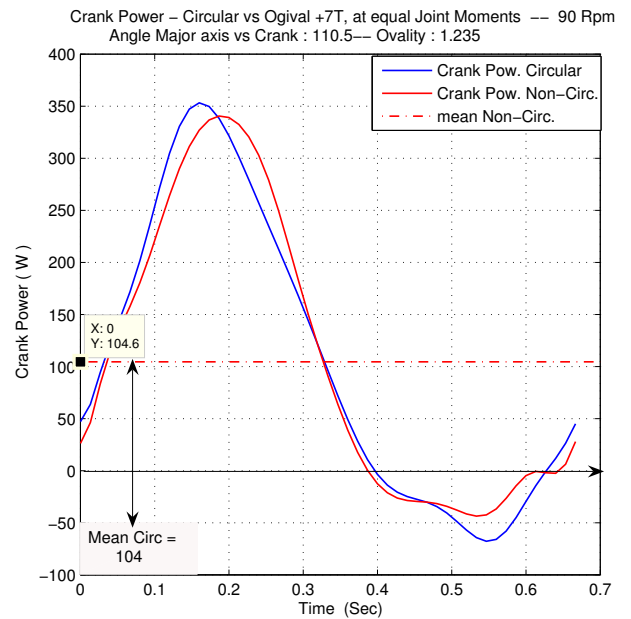
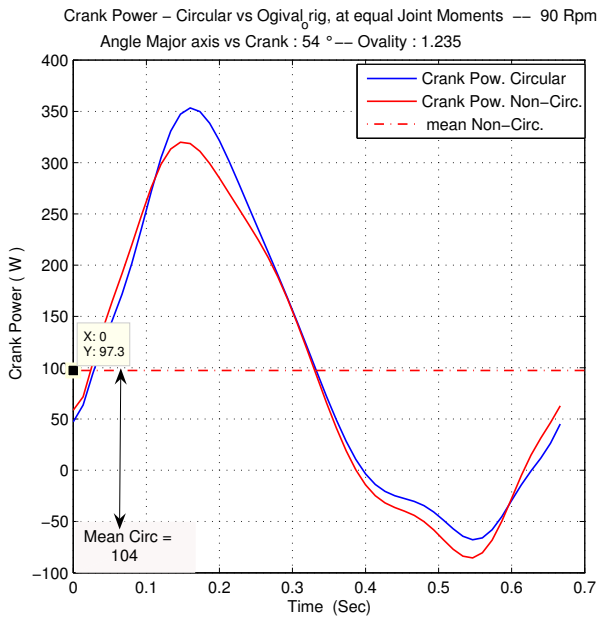


Ogival shape
Original Orientation

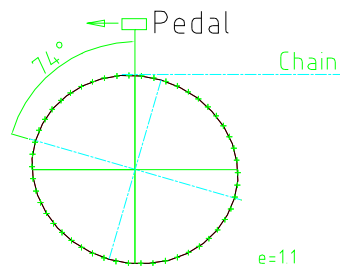
Ogival Optimal 110.5°



Ogival shape
Optimal Orientation

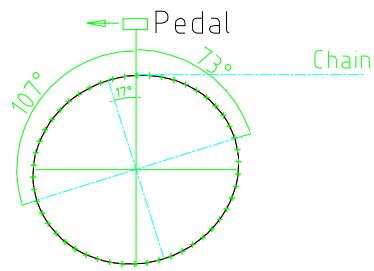


Q-Ring original 74°

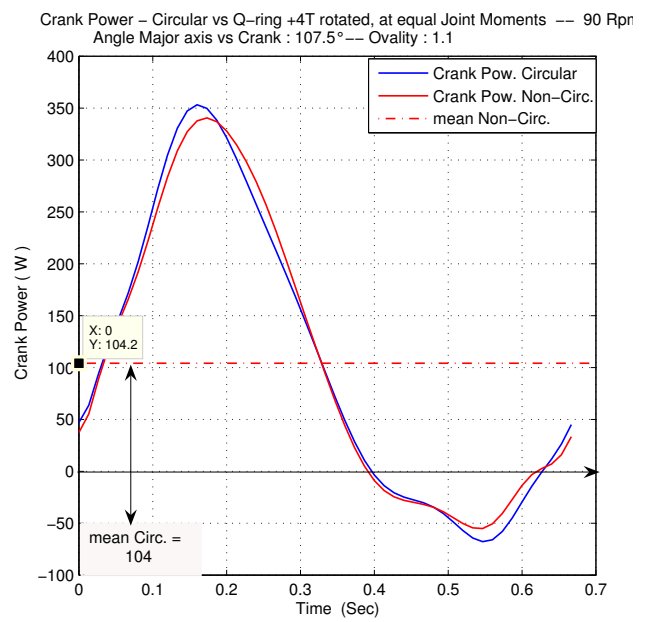
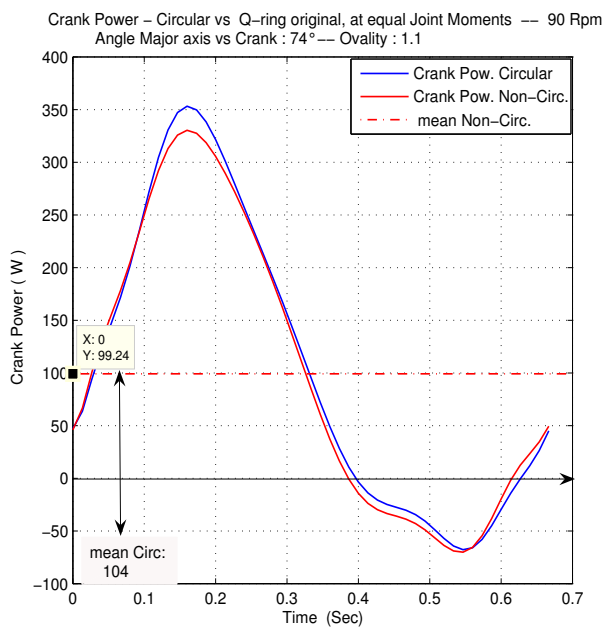


Q-Ring shape
Original Orientation

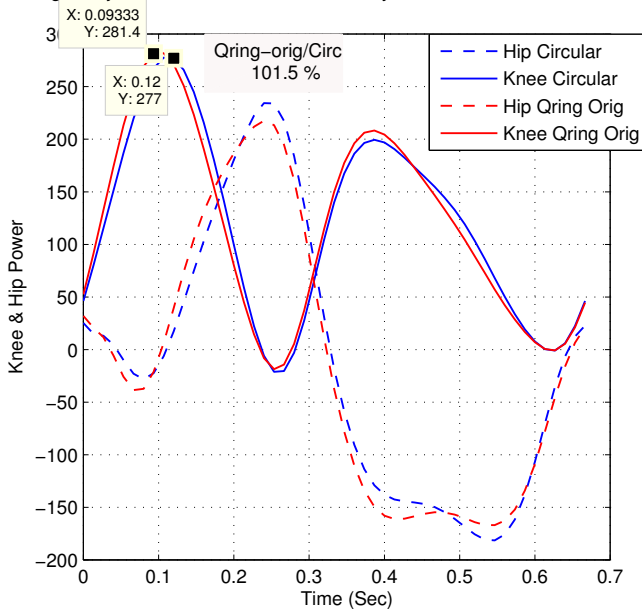
Q-Ring optimal 107.5°



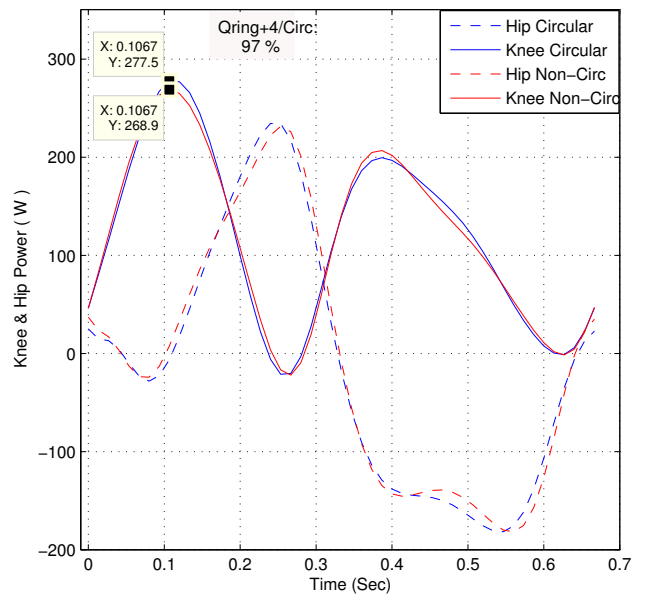
Q-Ring shape
Optimal Orientation



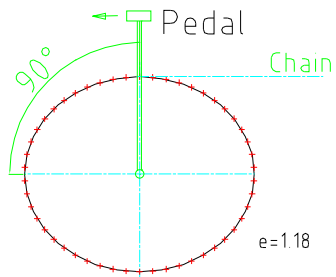
Knee & Hip power Circular vs Q-ring original, at equal Crank Power — 90 Rpm
Angle Major axis vs Crank : 74° — Ovality: 1.10



Knee & Hip power Circular vs Q-ring +4T rotated, at equal Crank Power — 90 Rpm
Angle Major axis vs Crank : 107.5° — Ovality: 1.10

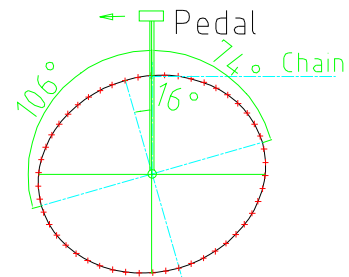


Ovum-118 Original 90°

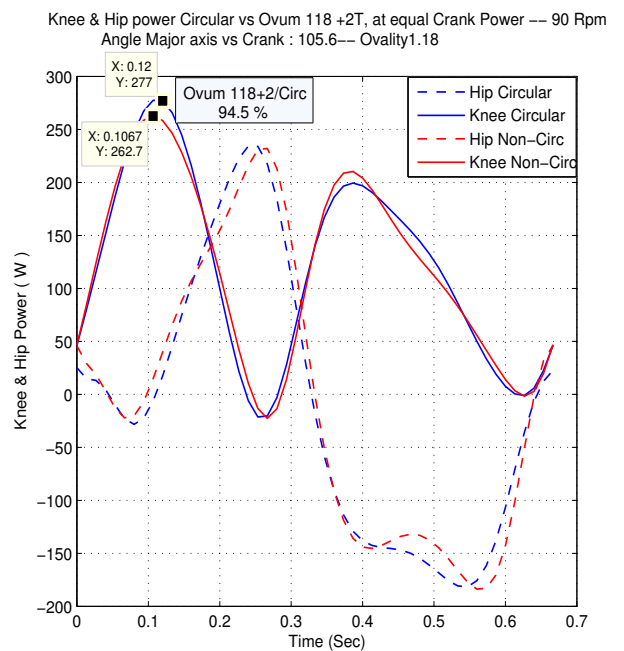
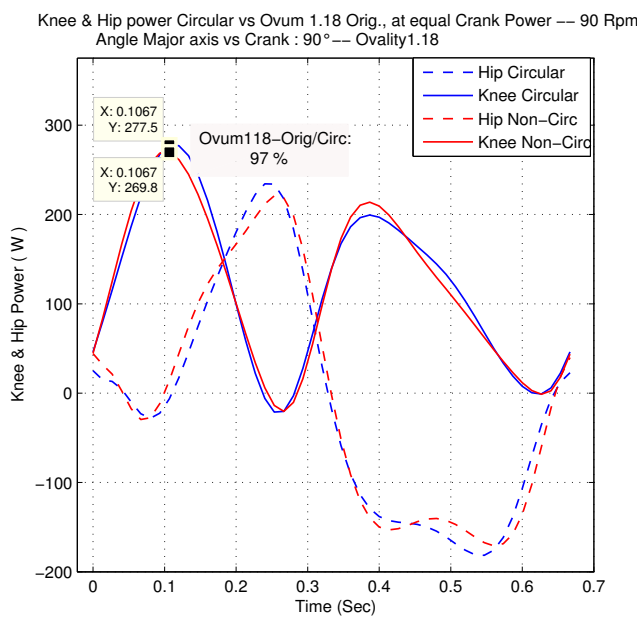
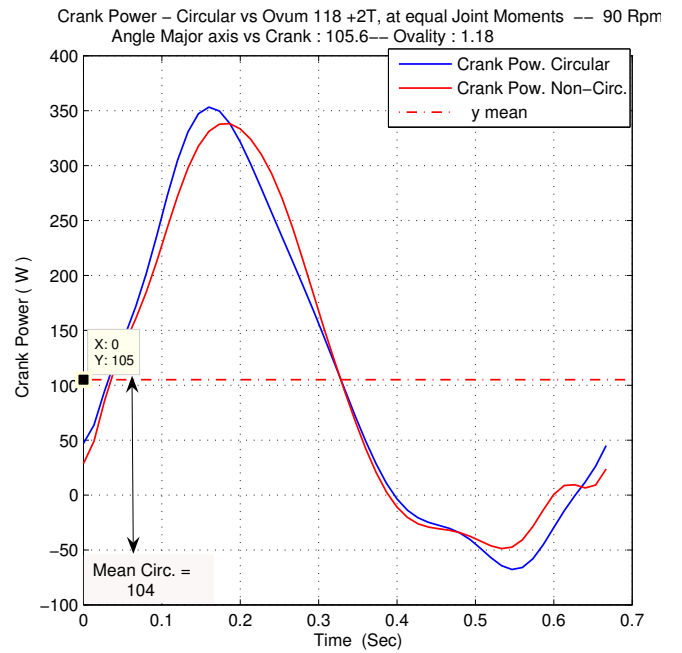
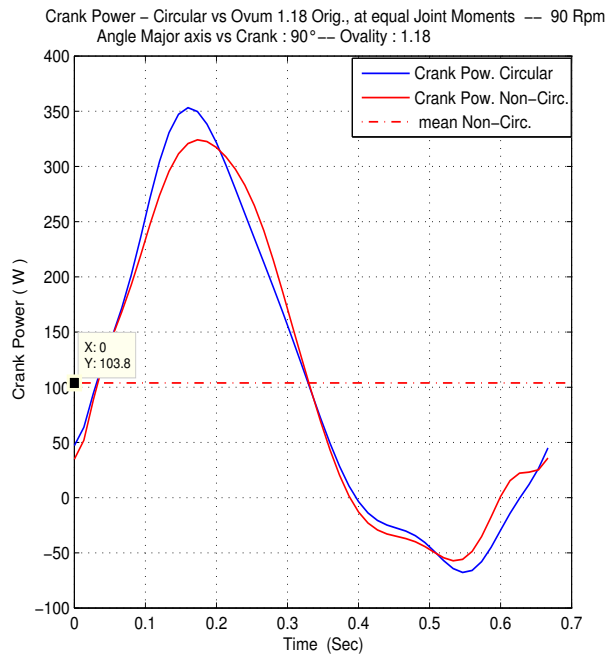


Ovum_1.18 shape
original Orientation

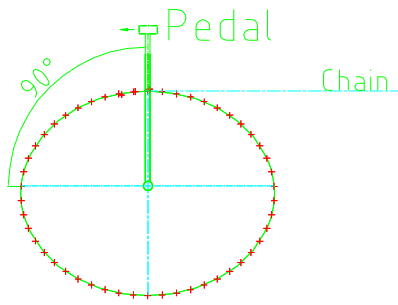
Ovum 118 Optimal 106°



Ovum_1.18 shape
Optimal Orientation

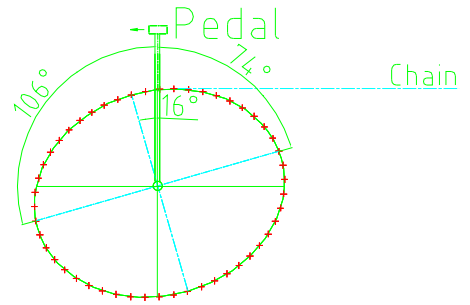


Ovum-124 Original 90°

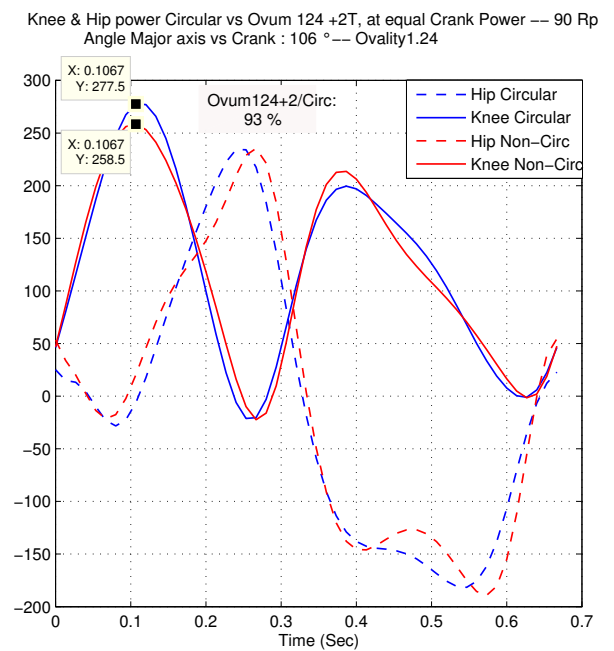
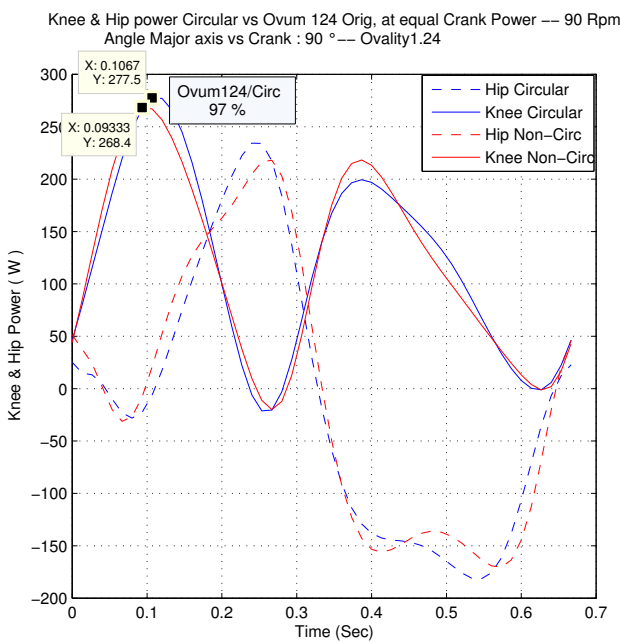
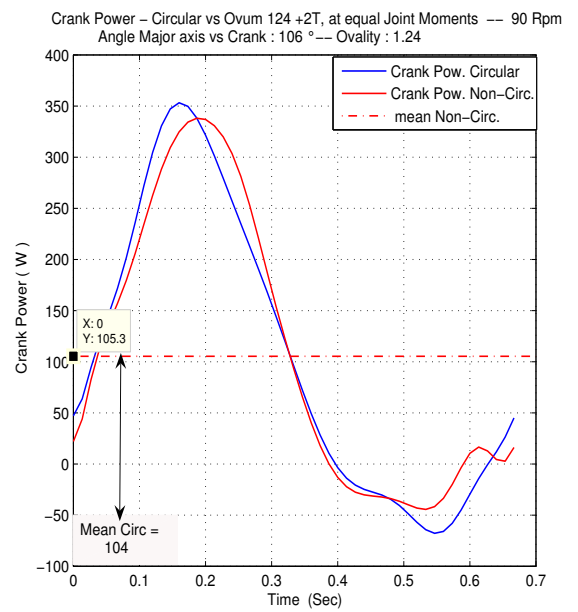
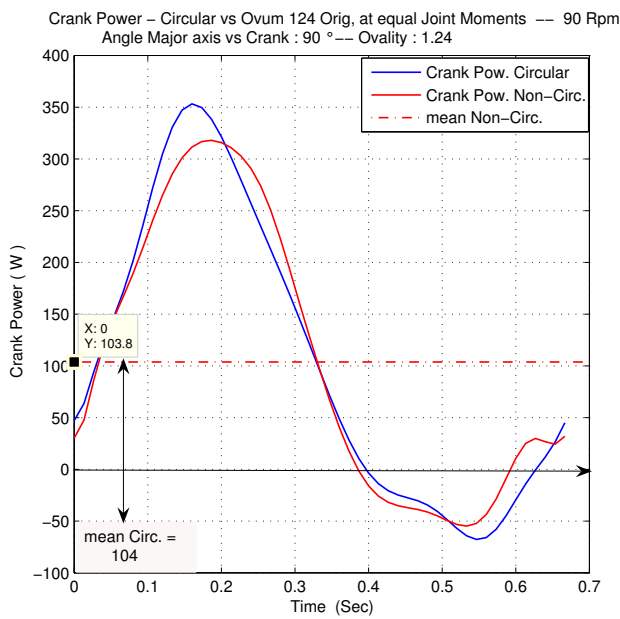


Ovum 124 shape
Original Orientation $e = 1.235$

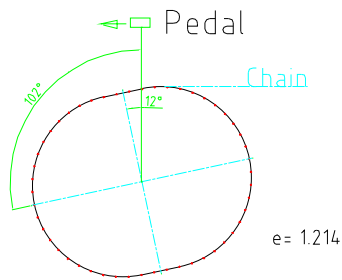
Ovum-124 Optimal 106°



Ovum 124 shape
Optimal Orientation

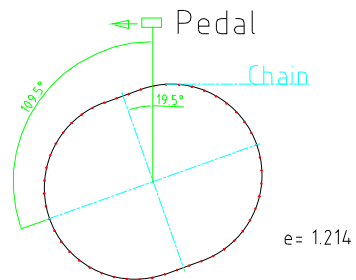


Pochlopek Original 102°



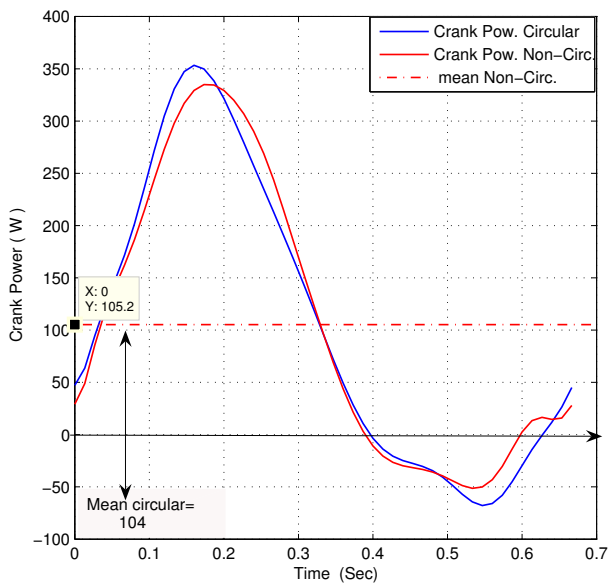
Polchlopek shape
Original Orientation

Polchlopek optimal 109.5°

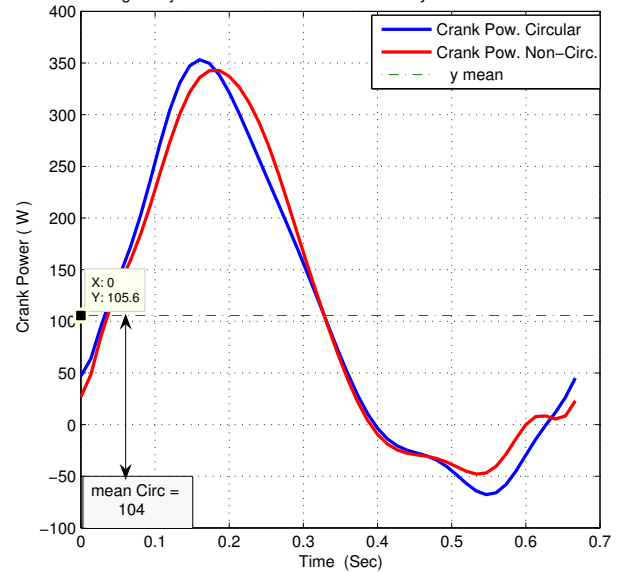


Polchlopek shape
Optimal Orientation

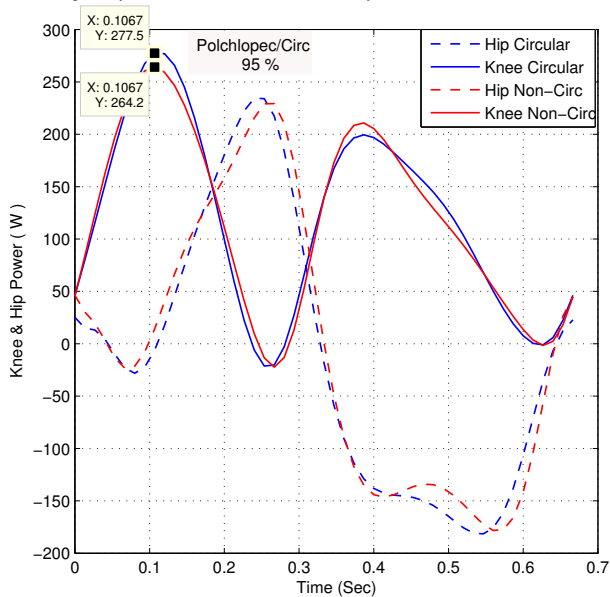
Crank Power – Circular vs Polchlopek Orig., at equal Joint Moments --- 90 Rpm
Angle Major axis vs Crank : 102° --- Ovality : 1.214



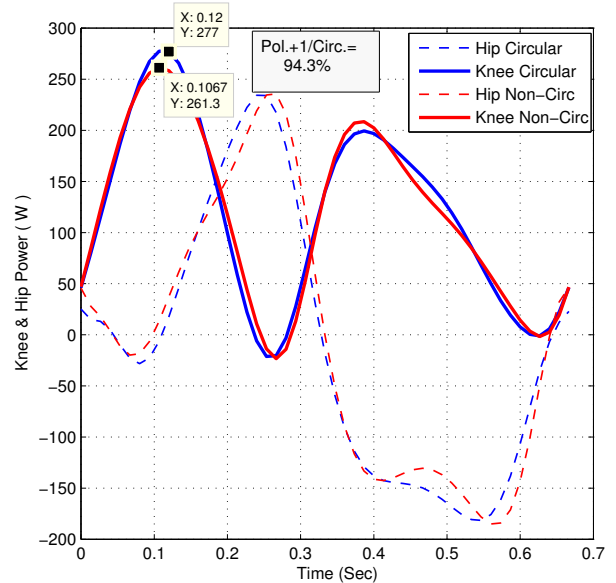
Crank Power – Circular vs Polchlopek+1T, at equal Joint Moments --- 90 Rpm
Angle Major axis vs Crank : 109.5° --- Ovality : 1.214



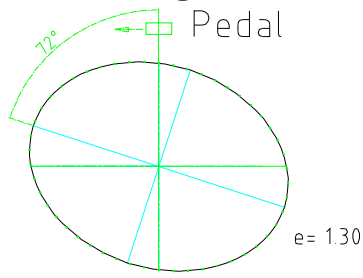
Knee & Hip power Circular vs Polchlopek Orig., at equal Crank Power --- 90 Rpm
Angle Major axis vs Crank : 102° --- Ovality : 1.214



Knee & Hip power Circular vs Polchlopek+1T at equal Crank Power --- 90 Rpm
Angle Major axis vs Crank : 109.5° --- Ovality 1.214

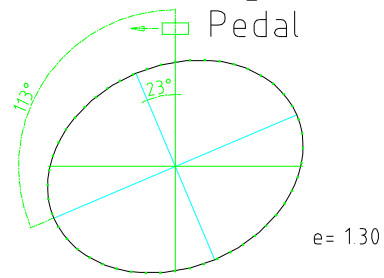


Rasmussen Original 73°



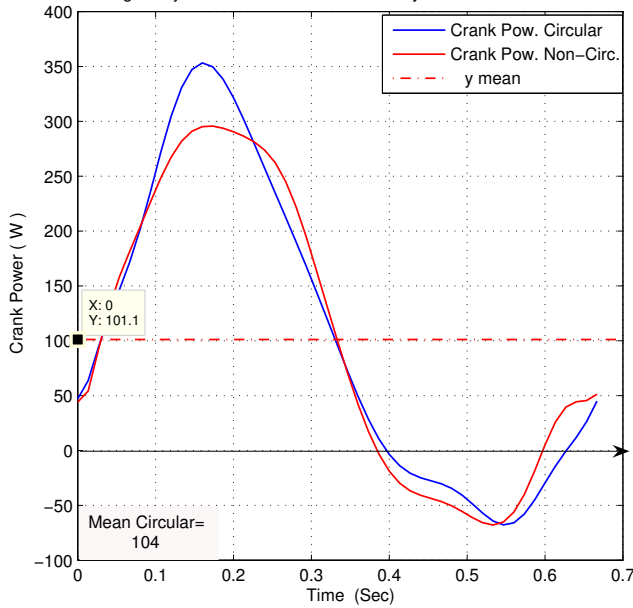
Rasmussen shape
Original Orientation

Rasmussen optimal 113°

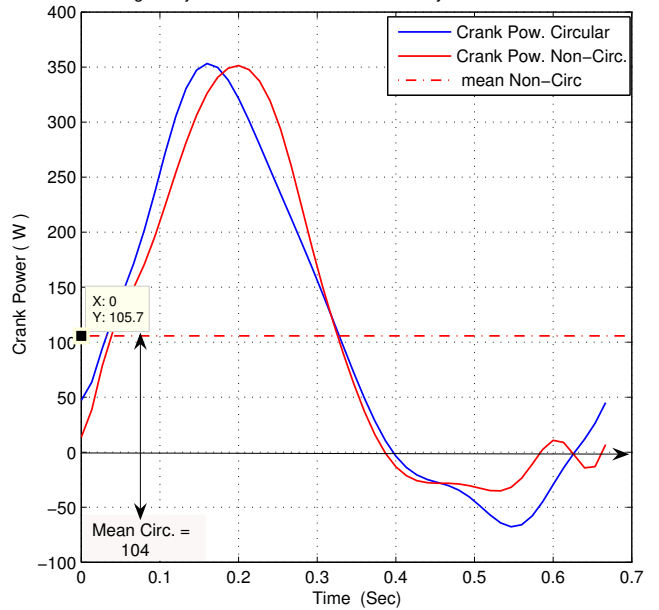


Rasmussen shape
Optimal Orientation

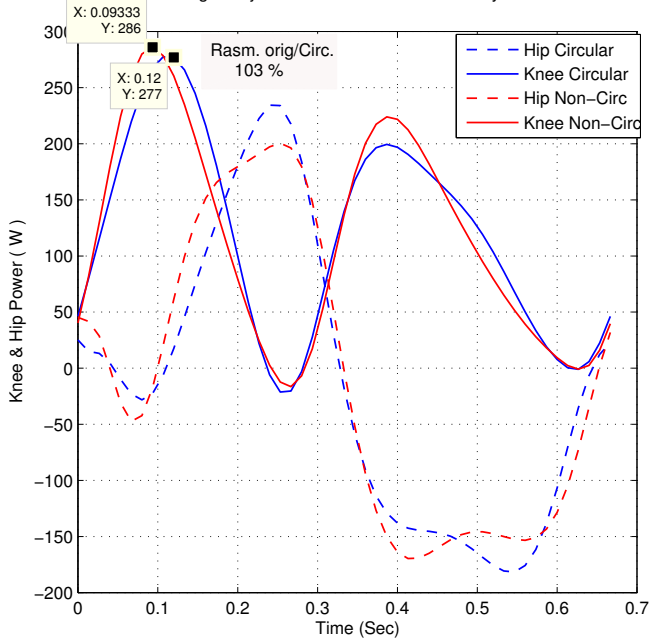
Crank Power – Circular vs Rasmussen Orig., at equal Joint Moments --- 90 Rp
Angle Major axis vs Crank : 72° --- Ovality : 1.30



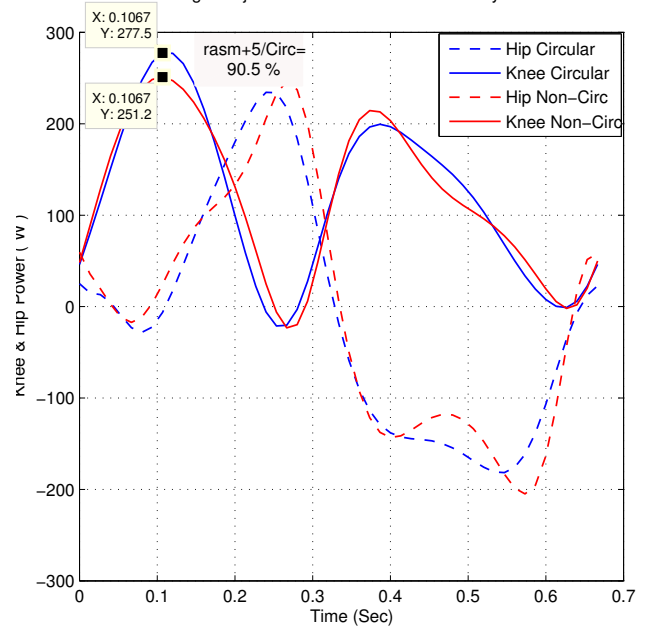
Crank Power – Circular vs Rasmussen+5, at equal Joint Moments --- 90 Rp
Angle Major axis vs Crank : 113° --- Ovality : 1.30



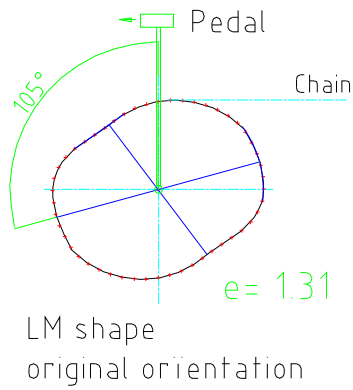
Knee & Hip power Circular vs Rasmussen Orig. at equal Crank Power --- 90 Rpm
Angle Major axis vs Crank : 72° --- Ovality1.30



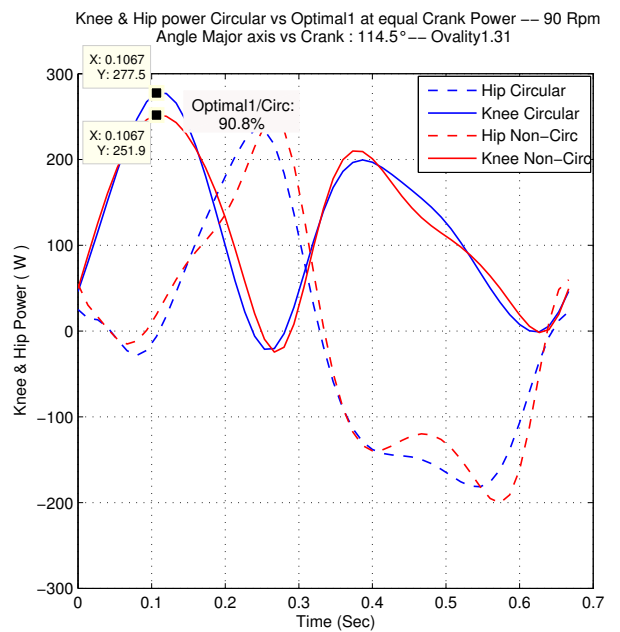
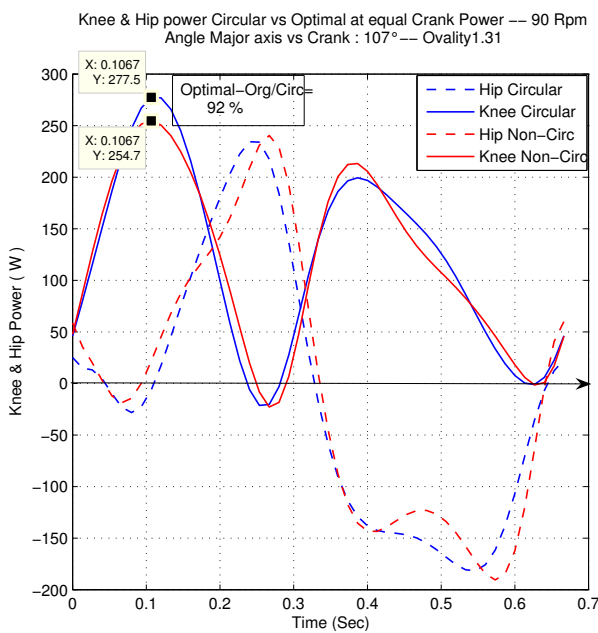
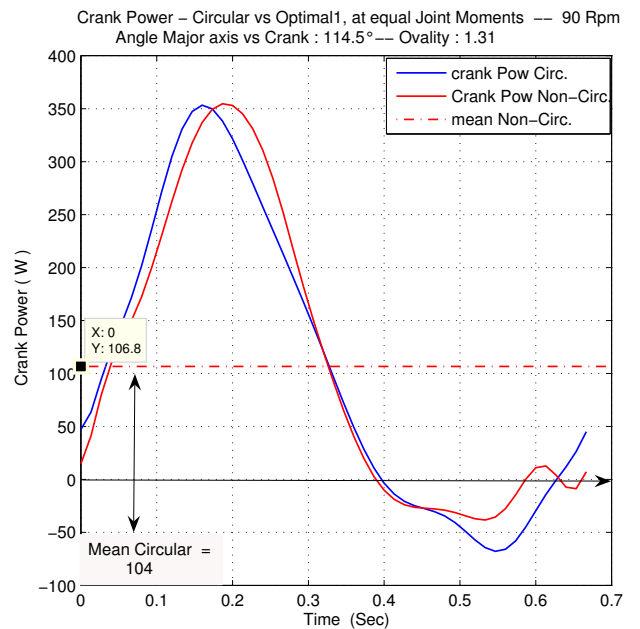
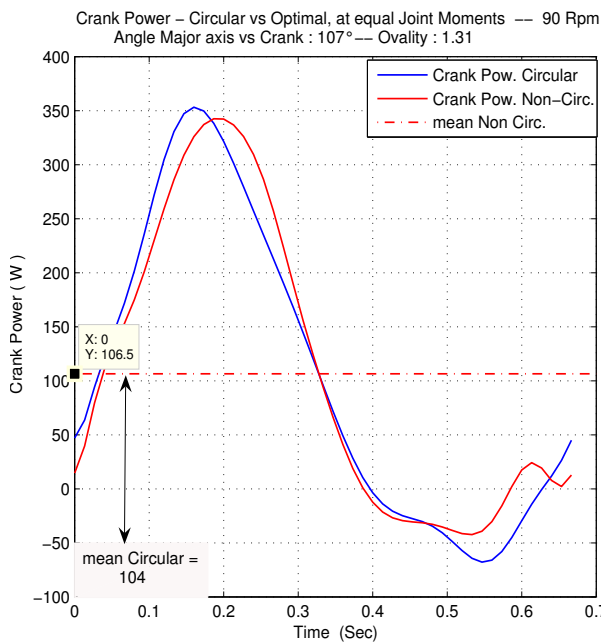
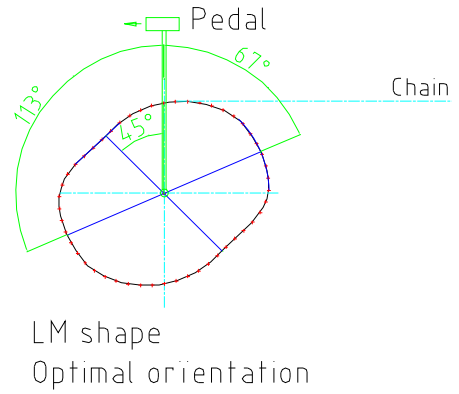
Knee & Hip power Circular vs Rasmussen+5 at equal Crank Power --- 90 Rp
Angle Major axis vs Crank : 113° --- Ovality1.30



LM-Super - Original 107°



LM-Super - Optimal 114.5°



6.2. Overview of results

| Non Circular Chainring shapes | Ratio Major versus Minor Axis | Angle Major Axis versus Crank | Peak Knee Power Extensors for same given Crank Power % difference vs Circular | Crank Power for same given Joint Moments % difference vs Circular |
|-------------------------------|-------------------------------|-------------------------------|---|---|
| Biopace | 1.04 | -8° | +1 % | -0.2 % |
| Hull Original | 1.55 | 90° | -5.5 % | +0.58 % |
| Hull Optimal | 1.55 | 107.5° | -11.8 % | +3.3 % |
| O.symmetric Original | 1.215 | 78° | -1.5 % | -0.67 % |
| O.symmetric Optimal | 1.215 | 117° | -7.5 % | +2.5 % |
| Ogival Original | 1.235 | 54° | +7 % | -6.4 % |
| Ogival Optimal | 1.235 | 110.5° | -6.5 % | +0.4 % |
| Ovum-118 Original | 1.18 | 90° | -3 % | -0.2 % |
| Ovum-118 Optimal | 1.18 | 106° | -5.5 % | +1.0 % |
| Ovum-124 Original | 1.24 | 90° | -3 % | -0.2 % |
| Ovum-124 Optimal | 1.24 | 106° | -7 % | +1.25 % |
| Polchlopek Original | 1.214 | 102° | -5 % | +1.15% |
| Polchlopek Optimal | 1.214 | 109.5° | -5.7 % | +1.54 % |
| Rasmussen Original | 1.30 | 72° | +3 % | -2.8 % |
| Rasmussen Optimal | 1.30 | 113° | -9.5 % | +1.6 % |
| Q-Ring Original | 1.10 | 74° | +1.5 % | -4.58 % |
| Q-Ring Optimal | 1.10 | 107.5° | -3 % | +0.2 % |
| LM-Super Original | 1.31 | 107° | -8 % | +2.4 % |
| LM-Super Optimal | 1.31 | 114.5° | -9.2 % | +2.7 % |

Ranking

| | | | | |
|---------------------|-------|--------|---------|---------|
| Hull Optimal | 1.55 | 107.5° | -11.8 % | +3.3 % |
| LM-Super Optimal | 1.31 | 114.5° | -9.2 % | +2.7 % |
| O.symmetric Optimal | 1.215 | 117° | -7.5 % | +2.5 % |
| Rasmussen Optimal | 1.30 | 113° | -9.5 % | +1.6 % |
| Polchlopek Optimal | 1.214 | 109.5° | -5.7 % | +1.54 % |
| Ovum-124 Optimal | 1.24 | 106° | -7 % | +1.25 % |
| Ovum-118 Optimal | 1.18 | 106° | -5.5 % | +1.0 % |
| Ogival Optimal | 1.235 | 110.5° | -6.5 % | +0.4 % |
| Q-Ring Optimal | 1.10 | 107.5° | -3 % | +0.2 % |
| Biopace | 1.04 | -8° | +1 % | -0.2 % |

7. Concluding remarks

Relying on a mathematical model, a biomechanical comparison was made between a circular and different non-circular chainring designs.

The mathematical model is partly based on literature study but is also based on own developments and new insights.

Especially the methodology to accurately measure the crank angle velocity is new and has, to the authors' knowledge, not been applied before. This accurate measuring method is however of paramount importance with non-circular profiles and is the necessary basis for reliable input data and hence a guarantee for reliable output results. Also the choice of the two criteria to compare performances - circular versus non-circular chainring – is new and has, to the authors' knowledge, not been applied before.

The results of this biomechanical study indicate clearly that (criterion 1) for equal instantaneous crank power for both circular and non-circular, the peak joint power loads can be influenced favourably or unfavourably by using non-circular chainring designs.

This is a purely (bio)-mechanical issue.

For equal instantaneous joint moments (criterion 2) for both, circular and non-circular chainring designs, the model calculates differences in total crank power efficiency and in peak power loads on the joints.

The results for both criteria are mostly concurrent.

Some non-circular chainring profiles are undeniably better than other designs and perform clearly better than circular ones, at least according to the criteria considered in this paper.

An analysis of the test results indicates clearly that three geometric parameters are important for optimal design of a non-circular chainring, namely the ovality, the crank orientation and the shape.

A balanced combination of these geometric parameters should result in the most optimal non-circular chainring compared to circular.

The most optimal solution would mean:

-the lowest peak power load in the joints, given the same crank power development (criterion 1)

-the highest crank power efficiency, combined with the lowest peak power load in the extensor joint muscles of knee and hip, given the same joint moments (criterion 2)

for both circular and non-circular.

A first important finding of the study is that a minimum ovality is needed to be able to yield attractive power efficiency rates. The results also show that a positive correlation exists between the degree of ovality and the attainable crank power efficiency percentage compared to circular. A second interesting conclusion is that for a specific non-circular chainring, peak power loads on the joints and the crank power efficiency can be adapted continuously by changing the crank orientation versus the major axis of the oval.

However, the results of the biomechanical study show clearly that, in many cases, ‘advantages’ and ‘disadvantages’ are inseparable. Indeed, by increasing the crank angle versus the major axis, for criterion 1 and criterion 2, it becomes apparent that:

- peak power load on both knee joint muscle groups, extensors and flexors, is decreasing, whereby:
 - peak power load of the extensors decreases to a minimum at optimal crank angle orientation. This minimum is below circular chainring peak power load.
 - peak power load of the flexors is mostly above circular chainring peak power load.
- peak power load on both hip joint muscle groups, extensors and flexors, is increasing, whereby:
 - peak power load of the extensors is mostly below circular chainring peak power load.
- crank power efficiency index is increasing to a maximum in the area of optimal crank orientation.

For the knee joints, experience shows that the extensor muscles are an important restricting factor. Overloading the knee extensor muscles frequently leads to knee injuries.

As a consequence, when searching for an optimal crank angle, given the geometry of the non-circular, it makes sense to aim for a minimization of the peak power load in the extensors of the knee joint, to try to maximize the crank power efficiency and to keep an acceptable peak power load on the knee flexor and on both types of hip joint muscles.

For all the investigated non-circular chainrings the above defined ‘optimal crank angle versus major axis’ falls in the range of 110° to 120°.

In case the crank is optimal oriented and the major axis of the non-circular chainring is vertical then we see the crank arm roughly perpendicular on the seat tube direction (“rule of thumb”).

This means that in such a position, the angular velocity of the crank is minimal (highest gear).

Assuming the optimal crank orientation, the ‘academic’ **Hull Oval** may be considered as being the best performing non-circular chainring but will most probably be problematic for practical use. The extreme ovality may cause front derailleur problems. This non-circular chainring was designed to test the hypothesis that the related angular velocity profile serves to effectively reduce internal work (pedalling rates 80-100 rpm) compared to constant angular velocity pedalling (circular chainring).

The results of the **LM-Super Oval** with optimal crank position support completely earlier findings of Rankin and Neptune (2008): ovality of about 30 % is needed for a crank power increase of nearly 3% at 90 rpm compared to a conventional circular chainring.

O.symmetric-Harmonic is the best performing commercially available non-circular chainring when the crank is oriented in the optimal position.

The ‘academic’ **Rasmussen Oval**, although having an ovality of 30% shows significantly weaker crank power gain compared to the LM-S Oval and the O.symmetric-Harmonic. The professor Rasmussen design is a result of an optimization process using the 3-D software AnyBody to find the chainring shape that minimizes the maximum muscle activation. Indeed, the reduction of peak knee power in e.g. the extensor muscles is significant (-9.5%), but probably leads to a lower than expected increase in crank power.

The **Polchlopek Oval**, although ‘comparable’ to the O.symmetric-Harmonic, performs much weaker than this last one (if crank optimal). Both non-circular chainwheels have the same ovality, the same optimal crank orientation and both have two ‘circle segments’ bridged by two ‘flat teeth segments’.

However the centres of the O.symmetric circle segments are also the centre of the oval, whereas the centres of the Polchlopek circle segments are not, but located on the major axis.

It is quite remarkable that Edmond Polchlopek the inventor of the oval design, was almost intuitively able to develop a non-circular chainring with a crank orientation very close to optimal.

For both **OVUM** ellipses (ovality 1.018 and 1.024) at optimal crank orientation, the impact of the ovality on the bio-mechanical results are clearly illustrated: higher ovality causes better performances.

It is clear that the **Ogival** was released onto the market with a completely wrong crank orientation. The mathematical model confirms the comments from users about relatively quick muscle fatigue in the knee joint. Re-orienting the extreme crank position into the optimal orientation improves the performances dramatically. New designs with improved (adaptable) crank orientation and other (higher) ovalities are recently available for the market but have not been studied yet.


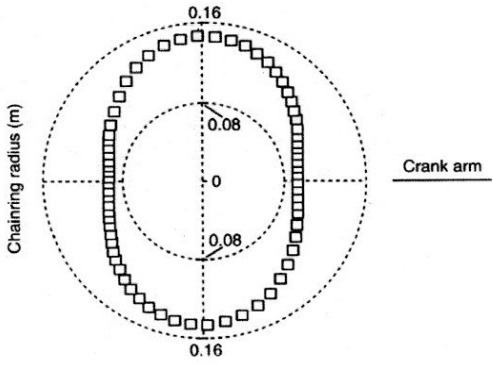
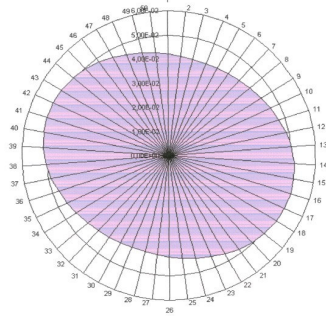




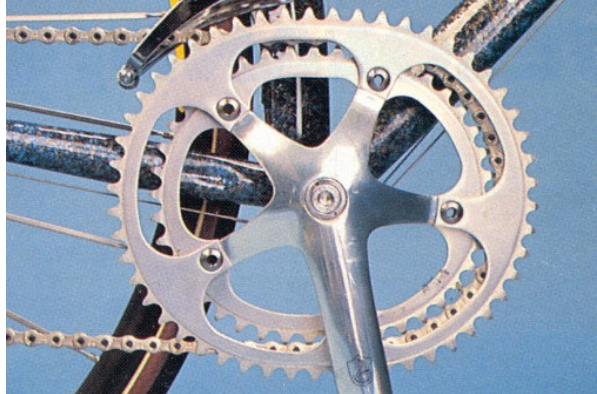
The **Q-Ring** is a brilliant example of excellent manufacturing workmanship but is first and foremost a compromise solution due to technical compatibilities and marketing reasons: ovality 1.10, slightly modified ellipse and crank at 74° . But as with all compromises, this oval is sacrificing most of its potential advantages. The problem of the Q-Ring is firstly its lack of ovality and secondly, the crank orientation. Even with the crank oriented in the optimal position the Q-Ring performances are disappointing and remain very weak. The mathematical model does not confirm the performance figures published by Rotor, neither in the crank orientation as advised by Rotor, nor in any other crank angle orientation.

Biopace (ovality: 1.04; crank -8.2°): this unusual crank orientation versus major axis brings the highest gear at the dead spots.

This low ovality Biopace design (1.04) behaves roughly as a circular chainring. Earlier models with higher ovality (e.g. 1.17 or 1.09 ...) gave an irregular and uncomfortable pedalling sensation. Users frequently reported knee problems. The solution implemented to reduce these problems was to reduce the ovality (1.04), in fact making it irrelevant. The mathematical model reveals the uncomfortable aspects and confirms what riders already concluded in the past: the Biopace is a totally wrong concept (crank arm oriented nearly parallel to the major axis).

As illustrated throughout the study, the mathematical model can be used as a tool for design optimization.

8. Overview of non-circular chainrings

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|--|--|
|  <p>O.symmetric-Harmonic</p> |  <p>Hull oval</p> |
|  <p>Rasmussen oval</p> |  <p>Q-Ring (Rotor)</p> |
|  <p>Biopace</p> |  <p>Ovum</p> |
|  <p>Ogival</p> |  <p>Polchlopek oval</p> |

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References

1. BARANI, D., COMMANDRE, F., and DIGION, A., The 'Harmonic Chainring': presentation and biomechanical characteristics. *Med Sport* 68: 77-81, 1994.
2. BOLOURCHI, F., and HULL, M.L., Measurement of rider induced loads during simulated bicycling. *Int. J. Sport Biomechanics* 1, 308-329, 1985.
3. BURKE, R., High Tech Cycling. *Human Kinetics*, 1996.
4. CORDOVA MARTINEZ, A., VILLA VINCENTO, G., SECO CALVO, J., and LATASA ZUDAIRE, I., Analysis of physiological and biomechanical effects of oval variable geared chainrings (Q-Rings) in comparison to conventional circular chainrings. Preliminary report on Q-Rings. *University of Valladolid, Department of Physiotherapy*, 2006.
5. DAPENA, J., A method to determine the angular momentum of a human body about three orthogonal axes passing through its center of gravity. *Journal of Biomechanics* 11, 251-256, 1978.
6. DEMPSTER, W. T., and GANGRAN, R.L., Properties of body segments based on size and weight. *American Journal of Anatomy*, 120, 33-54, 1967.
7. HULL, M.L., GONZALES, H. and REDFIELD, R., Optimization of pedaling rate in cycling using a muscle stress-based objective function. *Int. J. Sport Biomechanics* 4, 1-21, 1988.
8. HULL, M.L. and JORGE, M., A method for biomechanical analysis of bicycle pedalling. *J. Biomechanics* 18: 631-644, 1985.
9. HULL, M.L., WILLIAMS, M., WILLIAMS, K., and KAUTZ, S.A., Physiological response to cycling with both circular and non-circular chainrings. *Medicine and Science in Sports and Exercise* 24/ 1114-1122, 1992
10. OKAJIMA, S., Designing chainwheels to optimize the human engine. *Bike Tech* 2:1-7, 1983.
11. RANKIN, J.F., NEPTUNE, R.R., A theoretical analysis of an optimal chainring shape to maximize crank power during isokinetic pedaling. *Journal of biomechanics* 41, 1494-1502, 2008.
12. REDFIELD, R., and HULL, M.L., Prediction of pedal forces in bicycling using optimization methods. *J. Biomechanics* 19: 523-540, 1986.
13. REDFIELD, R., and HULL, M.L., On the relation between joint moments and pedalling rates at constant power in bicycling. *J. Biomechanics* 19: 317-329, 1986.
14. WHITSETT, C.F., Some dynamic response characteristics of weightless man. *AMRL Technical Documentary report* 63-18, 1963.

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